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Unclogging the Mathematics Pipeline Through Access to Algebraic Understanding

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Unclogging the Mathematics Pipeline Through Access to Algebraic Understanding

Section A: University of Connecticut and Yale University Algebra Pilot Research Study

(University of Connecticut Research Study: Schools 1 and 2 and Yale University Research Study: Schools 3, 4, and 5)

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ABSTRACT

The University of Connecticut and Yale University sites for the research study entitled Unclogging the Mathematics Pipeline Through Access to Algebraic Understanding involved grade 6 students for 30 hours of an after-school pilot research study in Algebra. Students who earned at least a B in mathematics were eligible for participation in the screening process, which included the Iowa Tests of Basic Skills, Mathematics Problem Solving and Data Interpretation (grade 8) subtest, and the Iowa Algebra Aptitude Test (grade 8).

The after-school pilot research study occurred for 10 weeks (1½ hours, twice a week). Teachers used *Connected Mathematics 2, Variables and Patterns,* a unit typically designed for grade 7 students. Of the 110 students assessed for the University of Connecticut research site, 73 participated in the program, with 30 students working with two teachers in School 1, and 43 students with three teachers in School 2.

[1] This project was originated at Yale University but completed at Tufts University when the Principal Investigator relocated.

A total of 90 students from 3 schools participated in the program for the Yale University research site, with 32 students working with 2 teachers in School 3, 31 students working with 2 teachers in School 4, and 27 students with 2 teachers in School 5. Schools 3 and 4 were in the same district. Students were thus divided into two groups at each school. Each group worked with one teacher who used technology or one teacher who did not use technology.

The pilot research study attempted to determine if involvement with above grade level curriculum would impact achievement, and attitude and interest toward mathematics. Student achievement in mathematics was assessed using four pre/post measures: Iowa Tests of Basic Skills, Mathematics Problem Solving and Data Interpretation subtest; Iowa Algebra Aptitude Test; *Connected Mathemati*cs Unit Test; and *Connected Mathematics* Unit Extended Test.

University of Connecticut Research Site Findings

All paired samples *t* tests on each achievement measure across and by schools yielded statistically significant differences.

Participation in the Algebra research study did not affect students' self-efficacy, and positive attitude and interest in mathematics. Students were positive about mathematics before and after their involvement in the after-school pilot research study. Their perceptions of the mathematics classroom practices in the after-school program indicated that the majority found the intensive Algebra program fun, interesting, and exciting. Many noted that the work differed from the regular classroom because it was more difficult. Yet, the students in this study found hard, difficult, and challenging work in Algebra to be fun and exciting.

Teachers and administrators shared their perceptions of teaching and learning mathematics. They recognized the importance of effective instruction in mathematics and were familiar with the characteristics of mathematically talented students. Challenging these students was important to the continuation of their learning.

Classroom observations provided a complete perspective on the research study as planned and as implemented. These observations confirmed teachers and students' adherence to the philosophy of the *Connected Mathematics Program,* and documented students' ability to understand and apply advanced-level knowledge and skills related to algebraic understanding. The dynamics within the classes were definitely determined by the teachers and students' commitment to learning how to think algebraically. Students mastered above grade level content and concepts and achieved representational fluency, which is the ability to solve problems using tables, graphs, words, or symbols. Algebraic reasoning prepares students for future accomplishments in mathematics, and the 73 students and their 5 teachers at the University of Connecticut pilot research schools were certainly successful in achieving the goals of this pilot research study.

Yale University Research Site Findings

Data analysis showed a gender x treatment interaction, with technology benefiting female students more than males.

Participation in the Algebra research study did not affect students' self-efficacy, and positive attitude and interest in mathematics. Students were positive about mathematics before and after their involvement in the after-school pilot research study. Their perceptions of the mathematics classroom practices in the after-school program indicated that there were mixed opinions, students liking some aspects of the program and not others. There was no consistent agreement on the difference between regular classroom practices and the after-school program.

Unclogging the Mathematics Pipeline Through Access to Algebraic Understanding

Section A: University of Connecticut and Yale University Algebra Pilot Research Study

(University of Connecticut Research Study: Schools 1 and 2 and Yale University Research Study: Schools 3, 4, and 5)

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EXECUTIVE SUMMARY

Too few identified gifted and potentially gifted students are exposed to concepts and competencies that will unclog the mathematics pipeline through access to algebraic understanding. Unclogging the Mathematics Pipeline Through Access to Algebraic Understanding involved grade 6 students in Algebra lessons for 30 hours of after-school instruction at 2 schools supported by the University of Connecticut and 3 schools with Yale University.

This pilot research study attempted to determine whether varying the form in which mathematical material is presented and adding an after-school component creates greater equity of opportunity for students to improve their mathematical performance and to increase their self-efficacy and positive attitudes toward and interest in mathematics. The quantitative and qualitative findings for Section A of this research report focus on the schools involved with the University of Connecticut and Yale University only.

University of Connecticut

Grade 6 students were screened for participation in the Algebra research study. All students with at least a B in mathematics were invited to take the grade 8 Iowa Tests of Basic Skills (ITBS) in Mathematics Problem Solving and Data Interpretation subtest and the grade 8 Iowa Algebra Aptitude Tests (IAAT). Of the 110 students screened, 73 qualified students chose to participate. Teachers implemented the *Connected Mathematics Program (CMP), Variables and Patterns*, which promotes reasoning and communicating proficiently in mathematics.

In School 1, the majority of students were African American (89.2%) and the majority in School 2 were White (84.5%). There were 30 participants in School 1 with 2 teachers, and 43 participants in School 2 with 3 teachers. An overview of the quantitative and qualitative findings for the University of Connecticut schools follows:

- Achievement data were analyzed using paired samples *t* tests across and by schools. There were statistically significant differences between pre and posttests on the ITBS, Mathematical Problem Solving and Data Interpretation subtest; IAAT; *Connected Mathematics* Unit Test; and the *Connected Mathematics* Extended Unit Test.
- The pre/posttest differences on the ITBS, Mathematics Problem Solving and Data Interpretation subtest indicated a 17-point gain across schools. The mean gain for School 1 students was 19.7 points, and the mean gain for School 2 students was 15.67.
- On the IAAT and the *Connected Mathematics* Unit Tests, students gained 2-3 points on every measure across and by schools.
- Students' attitudes toward mathematics were not significantly different on a pre/post basis. Students were very positive about math before and after their involvement in the after-school Algebra pilot research study. Attitudes Toward Mathematics (Tapia & Marsh, 2004) presents a mean score of 137.36 for high school students. The mean score for grade 6 students involved in the Algebra research study was over 160.
- Students completed a questionnaire at the end of the program and offered these words to describe the Algebra program: fun, exciting, awesome, challenging, educational, and worthwhile.
- When asked to recall their own school experiences about learning Algebra, 3 participating teachers remembered having a poor teacher who taught from the book, used a skill and drill instructional approach, and did not use a variety of activities to teach mathematical concepts. Two respondents commented that their teachers were fun, enthusiastic, and appeared to have a passion for mathematics.
- Administrators believed that effective mathematics teachers should engage students in math, make math lessons fun, teach math concepts in multiple ways, and serve as a learning and math role models.
- When asked if high potential math students are easy to identify, one administrator suggested using achievement data and "put energy into kids

who need to be identified." Another administrator commented that is was not always easy because high potential math students may be non-compliant and they don't conform to the steps/process.

Students and teachers were receptive to their involvement in the Algebra pilot research study. The grade 6 students spent an additional 3 hours per week to explore words, tables, and graphs and how these approaches led to algebraic understanding. The selected curriculum, *Connected Mathematics 2, Variables and Patterns,* provided a format and guide. However, the dynamics within the classes were definitely determined by the teachers and students' commitment to learning how to think algebraically. Students mastered above grade level content and concepts and achieved representational fluency, which is the ability to solve problems using tables, graphs, words, or symbolic representations. Algebraic reasoning prepares students for future accomplishments in mathematics, and the 73 students and their 5 teachers at the University of Connecticut research schools were certainly successful in achieving the goals of this pilot research study.

Yale University

To compare the pre/posttest differences across schools and across interventions, paired samples *t* tests were conducted on each matched set of achievement data: ITBS pre/post; IAAT pre/post; and CMP pre/post (12 items); CMP pre/post (15 items). For the overall population examined, ITBS scores dropped slightly from pre to posttest, while the IAAT and CMP (12 and 15 items) scores all increased.

In addition to examining the overall achievement scores, analyses of the results across treatment type were performed. The mean difference indicated increases in IAAT and CMP (12 and 15), while the ITBS scores remained approximately the same, in both the technology (treatment) and no additional technology (control) cases. The gains were not significantly different in either treatment case.

Further analyses resulted in paired samples *t* tests being performed where treatment type and gender were both considered. The changes in the mean IAAT, CMP 12, and CMP 15 pre and post scores were different for female and male participants in the two conditions. The presence of technology (treatment) increased the mean female scores more than the mean male scores, while the no additional technology condition (control) resulted in a greater rise in mean male scores than mean female scores. The results for the ITBS assessment were different than the other three assessments. The mean ITBS scores of females in the technology condition and males in the control condition actually decreased.

Students completed the Attitudes Toward Mathematics (Tapia) survey on a pre and posttest basis to help determine if involvement with after-school pilot research study affected students' self-efficacy and positive interest and attitudes in mathematics. The score for the instrument was the sum of all ratings. The mean pretest score for all students participating in the Yale Algebra research study was 139.89, and the mean posttest score was 141.33.

The data were also analyzed by condition and gender. There were no significant changes between pre and posttest attitudes toward mathematics in either condition or gender.

The results that proved interesting occurred when analysis was performed across gender and condition together. Females in the control group showed a small decrease in math attitude, while male's math attitude increased by 6 points in the control group. Meanwhile, the mean math attitude score of females in the treatment group increased by nearly 3 points and males in the treatment group decreased by nearly 3 points.

Participation in the Algebra research study did not affect students' self-efficacy, and positive attitude and interest in mathematics. Students were positive about mathematics before and after their involvement in the after-school pilot research study. Their perceptions of the mathematics classroom practices in the after-school program indicated that there were mixed opinions, students liking some aspects of the program and not others. There was no consistent agreement on the difference between regular classroom practices and the after-school program.

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Part I: Introduction

Many students do not see how procedures used to solve equations or simplify algebraic expressions are based on computations in arithmetic. Some regard Algebra as a study of the x, y, and z letters at the end of the alphabet. They have not viewed Algebra as a tool for analytical thinking or as a support for all levels of mathematics. Too few identified gifted and potentially gifted students are exposed to concepts and competencies that will unclog the mathematics pipeline through access to algebraic understanding.

Unclogging the Mathematics Pipeline Through Access to Algebraic Understanding involved grade 6 students in Algebra lessons for 30 hours of after-school instruction at 2 schools supported by the University of Connecticut and 3 schools with Yale University. Students were assigned to Intervention 1, which promoted the use of technology with graphing calculators (or Excel worksheets on computers) or Intervention 2 (non-technology). The following questions guided the pilot research study:

---

[2] This project was originated at Yale University but completed at Tufts University when the Principal Investigator relocated.

1. Do students who participate in the mathematics intervention with technology outperform non-technology group students on measures of mathematics achievement?
2. Do students who participate in the mathematics intervention with technology have higher self-efficacy, and positive attitudes and interest in mathematics than non-technology group students?
3. What are students' perceptions of the mathematics classroom practices in the after-school program?
4. What are teachers and administrators' perceptions of teaching and learning mathematics?

This pilot research study attempted to determine whether varying the form in which mathematical material is presented and adding an after-school component creates greater equity of opportunity for students to improve their mathematical performance and to increase their self-efficacy and positive attitudes toward and interest in mathematics.

Part II: Review of Literature

Mark Oliver
E. Jean Gubbins
University of Connecticut

Interest in studying the characteristics and capacities of mathematically talented students has continued to proliferate since Julian Stanley commenced his pioneering Study of Mathematically Precocious Youth in 1971. Talented math students have been reported as possessing unique learning characteristics that include an unusual quickness in learning, understanding, and applying mathematical ideas; an early ability to think and work abstractly; a capacity for developing creative solutions to mathematical problems; the ability to transfer learning to new, untaught mathematical situations; and a keen awareness of and interest in mathematics (Stanley, 1980; Waxman, Robinson, & Mukhopadhyay, 1996).

Identifying talented math students can be problematic, as definitions of mathematical giftedness vary (Mann, 2006). Some definitions may exclude students due to the emphasis on overt characteristics such as rapid computation skills. Mann (2006) asserted that this is the case for creatively talented math students, who are rarely identified by typical classroom assessment measures. Other groups that may not be identified as being talented at math include females (Stepanek, 1999) and minority students (Ford & Trotman, 2000; Konstantopoulos, Modi, & Hedges, 2001; Maker, 1996). Considering the issues associated with the identification of mathematically precocious youth, the population of talented math students may be larger and more diverse than some prevalence estimates (e.g., 3% as reported by Stanley & Benbow, 1982).

Broadened Conceptions of Giftedness

Since the 1970's several definitions of giftedness have been offered. Marland's report (1972) was the first federal definition and in subsequent years, some of the terminology changed. For the Javits Act of 1988, the earlier federal definition was modified. In 1993, with the release of the second federal report on the state of gifted and talented education, further revisions were made. Throughout The National Research Center on the Gifted and Talented (NRC/GT) research, we have supported the following definitions: federal definition based on the Jacob K. Javits Gifted and Talented Students Education Act and broadened definitions offered by Renzulli and Sternberg:

> **National Excellence Report**
> Children and youth with outstanding talent perform or show the potential for performing at remarkably high levels of accomplishment when compared with others of their age, experience, or environment.
>
> These children and youth exhibit high performance capability in intellectual, creative, and/or artistic areas, possess an unusual leadership capacity, or excel in

> specific academic fields. They require services or activities not ordinarily provided by the schools.
>
> Outstanding talents are present in all children and youth from all cultural groups, across all economic strata, and in all areas of human endeavor. (U.S. Department of Education, 1993, p. 26)
>
> **Renzulli's Definition**
> Giftedness consists of an interaction among three basic clusters of human traits—these clusters being above average general abilities, high levels of task commitment, and high levels of creativity. Gifted and talented children are those possessing or capable of developing this composite set of traits and applying them to any potentially valuable area of human performance. Children who manifest or are capable of developing an interaction among the three clusters require a wide variety of services that are not ordinarily provided through regular instructional programs. (Renzulli, 1978, p. 261)
>
> **Sternberg's Definition**
> Gifted children, according to Sternberg (1985, 2000), tend to excel in the metacomponents of intelligence: (a) recognizing the existence of a problem, (b) identifying the nature of the problem, (c) allocating resources to problem solution, (d) mentally representing the problem, (e) formulating strategies for solving the problem, (f) monitoring one's problem solving while it is being done, and (g) evaluating one's problem solving after it is done. These are modifiable skills.

These broadened definitions emphasize the modifiability of one's intelligence as they include traits, aptitudes, and behaviors that are to be developed and nurtured in the home and school. Achievement expectations influence what students experience in the school curriculum. Often these expectations become entrenched in challenging environments. All children need to be exposed to rigorous and challenging curriculum that guides them from what they currently know, understand, and are able to do to what they need to know, understand, and are able to do to be successful academically.

Status of Mathematical Knowledge, Skills, and Curriculum

Interest in the mathematical knowledge and skill attainment of talented math students has been discussed for many years. The National Council of Teachers of Mathematics (1980) commented that "outstanding mathematical ability is a precious societal resource" (p.18), however asserted that talented math students were the most neglected in terms of educational opportunities. This neglect may explain the relatively poor achievement of talented math students on international tests, such as the Trends in International Mathematics and Science Study (TIMSS). Sheffield (2006) reported that the performance of U.S. students in mathematical literacy and problem-solving was lower than the average performance for most OECD (Organization for Economic Co-operation and Development) countries, and that even the highest U.S. achievers were outperformed on average by their OECD counterparts. Other research (Schreiber, 2000) indicated that only 26% of high school seniors taking advanced mathematics in their sample (n = 2,349) scored above the international mean, which indicated that the most advanced students in the United States were not reaching a high level of mathematical attainment.

Curriculum reform was seen as a necessary step to improve the mathematical knowledge and skill level of U.S. students. Proponents of the "Back to Basics" movement, such as the National Mathematics Advisory Panel created by the Bush Administration in 2006, have stressed the need for drill and practice in basic computation (O'Brien, 2007). Critics have argued that the progressive curriculum reform of the 1980s and 1990s is to blame for the disappearance of rigor from school math curriculum (Center for Comprehensive School Reform and Improvement, 2006; Klein, 2007), most specifically course materials that were based upon the curriculum and evaluation standards developed by the National Council of Teachers of Mathematics (NCTM) in 1989 (Klein, 2007). Reports such as "A Nation at Risk" (National Commission on Excellence in Education, 1983) and "A Report on the Crisis in Mathematics and Science Education" (American Association for the Advancement of Science, 1984) suggest however that student attainment of mathematical competencies was a major problem prior to the publication of the 1989 NCTM standards.

As the math wars continue to be waged in terms of curriculum content and sequence for the general population of students (Newton, 2007; O'Brien, 2007), the issue of "What is being done for talented math students?" needs to be examined. With the dumbing down of curriculum and the lowering of academic standards due to the current emphasis placed on high-stakes testing (Renzulli, 2005), investigation of the most appropriate instructional methods to develop the talents of precocious math students is warranted. Miller (1990) stressed that no single instructional method is superior because the characteristics and needs of mathematically talented youth vary greatly.

A variety of instructional options, including enrichment and accelerative practices, have been described as being suitable for meeting the needs of gifted math students (Ysseldyke, Tardrew, Betts, Thill, & Hannigan, 2004). Enrichment programs can improve student motivation (Renzulli & Reis, 1997), however without accelerative components, enrichment alone may not be an adequate for developing the talents of mathematically gifted students (Kondor, 2007). Miller (1990) stressed that flexible pacing and advanced content should be key components of mathematics programs for gifted math students. Stanley (Brody & Stanley, 2005; Stanley, 1980; Stanley & Benbow, 1982) supported the notion of exposing talented math students to advanced content, and consistently advocated that such students should be provided with opportunities for acceleration. Accelerative options for gifted math students include compacted math courses, access to advanced level courses (e.g., Algebra and Calculus), and early entrance to or dual enrolment in an advanced level of school (Miller, 1990). Despite these recommendations, many gifted students do not receive instruction tailored to their unique needs (Archambault et al., 1993), and unfortunately many math programs for the gifted are poorly designed (Heid, 1983).

Ma (2000) analyzed the Longitudinal Study of American Youth database (grades 7-12) to determine the impact of coursework in pre-Algebra, Geometry, and Calculus on achievement and attitudes toward mathematics. There was a variable effect of coursework in secondary school, depending on the grade levels. Algebra had a significant impact on achievement for the early grades of high school; mathematics coursework did not have a similar impact on middle grades; and, in later grades in high school, every course affected achievement in mathematics. Essentially, more time and more coursework made a significant difference, after adjusting for student demographic characteristics.

Algebra Benefits Students

Gifted education researchers have recommended early exposure to advanced curriculum content for precocious youth (Brody & Stanley, 2005; Reis & Renzulli, 1992). Providing an early introduction to Algebra has been viewed as an essential step towards developing the capacities of talented math students (Stanley, 1980; Stanley & Benbow, 1982), however it has been argued that early exposure to Algebra concepts would provide benefits for all students (Choike, 2000; Kaput, 1995; Krebs, 2003). Blair (2003) asserted that, "in today's technological society, algebra has become a gatekeeper for citizenship and economic access. As the world has become more technological, the reasoning and problem solving that algebra demands are required in a variety of workplace settings" (p. 1). The U.S. Department of Labor (cited in Krebs, 2003) reported that "the number of mathematics courses taken during high school was the strongest predictor of earnings nine years after graduation" (p. 234). In addition to economic benefits for individual students and society (Sheffield, 2006), competencies in Algebra also improve the likelihood of college success (Choike, 2000).

The arguments made to introduce Algebra earlier in the school curriculum to improve college success, future earnings, and to maintain a competitive U.S. workforce (Sheffield, 2006), have stimulated interest regarding methods to increase rates of participation in Algebra courses and also how to improve algebraic understandings. The National Center for Educational Statistics (2005) reported that enrollment in Algebra courses has steadily improved since 1978, particularly for students who are Black, Hispanic, or female. The overall performance on national math assessments has also improved (National Center for Educational Statistics, 2007), however results for student performance specifically related to Algebra were not available. O'Brien (2007) reported that only 23% of students in California are proficient in Algebra I by the end of high school, however did not mention how many students participated in formal Algebra coursework. Despite the mixed findings from national assessments, the results from international measures such as TIMSS provide an impetus to investigate factors that may impede the development of algebraic reasoning.

For many students, "getting into college" is not a realistic goal unless they have experienced several prerequisites. For example, undergraduate applicants to the University of Connecticut must have completed a minimum of Algebra I, Algebra II, and Geometry. Educators understand the importance of algebraic thinking and the NCTM concurs. Kilpatrick, Swafford, and Findell (2001) state, "The formal study of algebra is both the gateway into advanced mathematics and a stumbling block for many students" (p. 419). As the president of the NCTM said, "I think most everybody recognizes the importance of algebra. It is a question of how they introduce it and when . . . " (cited in Blair, 2003, p. 1). The way some teachers teach and the way some students learn make it difficult for students to develop algebraic thinking and understanding. NCTM recommends that instructional programs from prekindergarten through grade 12 should enable all students to

- understand patterns, relations, and functions;
- represent and analyze mathematical situations and structures using algebraic symbols;
- use mathematical models to represent and understand quantitative relationships;
- analyze change in various contexts. (NCTM, 2003, p. 222)

Blair (2003) notes, "many students studying high school algebra don't see the procedures they use to solve equations or simplify expressions as based on the same properties that they used in arithmetic computation" (cited in Carpenter, Franke, & Levi, 2003, p. 1). The NCTM standards reinforce the approach to teaching Algebra in middle school. NCTM (2003) recommends that students

> learn algebra both as a set of concepts and competencies tied to the representation of quantitative relationships and as a style of mathematical thinking for formalizing patterns, functions, and generalizations. (p. 223)

Attitudes Towards the School Algebra Experience

Attitude towards math is one factor that may significantly influence students' attainment of mathematic competencies (Tapia & Marsh, 2004). Tapia (1996) asserted that the decline in math achievement may be due more to negative attitudes about math as opposed to what pedagogical approach or instructional material is used to teach the subject. Blair (2003) commented about the algebraic experience for learners, which illustrated the attitude of many students towards studying Algebra:

> I experienced algebra much like millions of other Americans—as an intensive study of the last three letters of the alphabet. I failed to grasp the importance of algebra—how it provides support for almost all of mathematics or to understand its power as a tool for analytical thinking. It was course I endured to get into college. (p. 1)

Tapia and Marsh (2004) identified four factors that contribute to the formation of math attitudes, those being: self-confidence, value, enjoyment, and motivation. Bandura (1986) asserted that confidence in one's own abilities (i.e., self-efficacy) was a critical component necessary for successful learning. Research (Pajares, 1996; Pajares & Graham, 1999) has indicated that gifted students possess stronger self-efficacy beliefs about mathematics, and experienced lower math anxiety than did regular education students.

McCoy (2005) studied the impact of demographic and personal variables on achievement in Algebra for grade 8 students, and determined that ethnicity, socioeconomic status, and attitudes affected mathematics scores. McCoy found that attitude scores decreased over time as students became more involved in Algebra. Wilkins and Ma (2003) also found a negative change in students' attitudes toward and beliefs about the social importance of mathematics as they progressed through middle and high school. In contrast, Higgins (1997) compared the effect of instruction in mathematical problem solving and middle school students' attitudes, beliefs, and abilities. Three classes of students received the problem solving intervention for 1 year and 3 classes continued with the traditional approach as the control group. Students in the problem-solving group persevered while solving mathematical problems, had more positive attitudes, and more advanced definition of mathematical understanding (Moses & Cobb, 2001).

Gilroy's study (2002) with 3 high schools emphasized the importance of students' attitudes toward mathematics because they ultimately influence students' motivation and achievement. Students with more positive attitudes toward mathematics were more inclined to enroll in additional coursework, which also influences future performance. To improve student learning in math and science, they must understand the "big ideas," which will allow them to apply their knowledge, skills, and understandings. Learning facts, solving problems, and taking tests do not necessarily result in a deep understanding of mathematical concepts.

Understanding students' perceptions about their mathematical knowledge, skills, and abilities may help unravel their potential impact on achievement. Determining the extent to which students' self-efficacy, and attitudes toward and interest in mathematics influences achievement is an important step in increasing the number of students who pursue and excel in mathematics.

Algebraic Understanding

Given the importance of studying Algebra, research concerning the most suitable teaching methods to enhance students' understandings of and positive attitudes towards Algebra should be conducted. Fostering the development of algebraic thinking is critical for those students who have the potential and commitment to transition from arithmetic to Algebra. They advocate the transformation of arithmetic activities and word problems with single numerical answers:

> [provide] opportunities for discovering patterns and making conjectures or generalizations about mathematical facts and relationships and justifying them. This can be as simple as encouraging children to discuss why they believe a mathematical statement or solution to a problem is correct. Blanton and Kaput suggest teachers use the following prompts as ways to extend student thinking:

- Tell me what you were thinking.
- Did you solve this in a different way?
- How do you know this is true?
- Does this always work? (cited in Blair, 2003, p. 2)

Blanton and Kaput (2003) noted that the classroom culture is integral to algebraic thinking. The classroom culture must value "students modeling, exploring, arguing, predicting, conjecturing, and testing their ideas, as well as practicing computational skills" (cited in Blair, 2003, p. 2). Briggs-Hale, Judd, Martindill, and Parsley (2006) determined that three prominent ideas add rigor to mathematics learning and assist to create a classroom climate to foster the development of mathematical thinking, those being: (a) encouraging problem solving; (b) developing math talk; and (c) emphasizing working together.

Problem solving has been purported to be a critical component for learning mathematics. When students have opportunities to explore their preconceptions and engage their own problem solving strategies, they are able to build new knowledge (National Research Council of the National Academies, 2005). Briggs-Hale et al. (2006) reported that good problem solving is fostered by problems that are interesting to and challenging for students, which encourage the development of thinking skills, and ultimately students' enthusiasm for learning.

Developing math talk involves students using mathematical language to express ideas, and share mathematical strategies and solutions between students and with the teacher. Briggs-Hale et al. (2006) found that students develop reasoning and metacognitive skills when communicating mathematically with one another. Emphasizing working together refers to the construction of a collaborative classroom environment where students communicate to solve problems together. By utilizing teaching practices that incorporate problem solving, math talk, and collaboration, teachers may construct learning environments that support students to develop complex mathematical thinking. Such environments would illustrate the transformation of "single-numerical answer" curriculum approaches to rich learning opportunities that emphasize the discovery of patterns, making of conjectures about mathematical facts, and mathematical relationships (Blanton & Kaput, cited in Blair, 2003, p. 2).

Specific Teaching Approaches

In some countries, students are introduced to Algebra for several years throughout their mathematics curriculum. The notion of Algebra for every student is a positive stance on expectations and educational attainments. However, offering a standard Algebra course to all is "virtually guaranteed to result in many students failing to develop proficiency in Algebra, in part because the transition is so abrupt" (Kilpatrick, Swafford, & Findell, 2001, p. 420). They recommend a different curriculum for Algebra in middle school:

> Teachers, researchers, and curriculum developers should explore ways to offer a middle school curriculum in which algebraic ideas are developed in a robust way and connected to the rest of mathematics. (p. 420)

After-school programs for gifted students can contribute to talent development by exposing them to advanced coursework and by cultivating social support between gifted students (Gardner et al., 2001; Olszewski-Kubilius, 2003). Many after-school programs were not, however, designed to provide academic challenge for students (Briggs-Hale et al., 2006), but instead were traditionally organized to provide remedial support or supervision for children whose parents were employed during the hours after school (Shumow, 2001). Given the current emphasis on increasing student achievement, many after-school programs now include learning activities that intend to develop the academic competencies of participating students (Briggs-Hale et al., 2006; Gardner et al., 2001; Shumow, 2001).

It may be perceived that participation in curriculum-based activities after school would produce growth in academic concepts and socialization for students, however a national evaluation study demonstrated that this might not be the case. In their evaluation of 7,500 after-school programs in rural and inner-city public schools, Dynarski et al. (2003) concluded that after-school programs had a limited influence on academic performance, no influence on feelings of safety, and negative influences on behavior. Case studies have shown however that after-school programs can have positive academic and social effects (Baldwin-Grossman, Walker, & Raley, 2001; Fleming-McCormick & Tushnet, 1996; Gardner et al., 2001; Posner & Vandell, 1994).

Considering the discrepancy in findings regarding program effectiveness, systematic evaluation of successful programs to establish essential aspects of after-school programs is warranted (Noam, Biancarosa, & Dechausay, 2003). Such inquiry may seek to establish what curriculum designs and teaching approaches for after-school programs are most effective for producing academic and social growth for students.

Understanding students' perceptions about their mathematical knowledge, skills, and abilities may help unravel their potential impact on achievement. Determining the extent to which students' self-efficacy, attitudes toward and interest in mathematics influences achievement is an important step in increasing the number of students who pursue and excel in mathematics.

Part III: Demographic Characteristics of Districts and Schools

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University of Connecticut Schools

The University of Connecticut implemented the pilot research study in 2 districts, with one school participating in each. Overviews of the demographic characteristics of the districts and schools are provided below.

District and School 1 Profile

According to the 2000 census, the district for School 1 has a population of 19,585 with a per capita income of $28,843. Of the adult population, 84% have earned a high school diploma. The district borders a major urban city and is home to many businesses, places of worship, and recreation areas. Throughout the district are private homes in neighborhoods with easy access to major highways. New homes, condominiums, and townhomes are now occupying land that once belonged to businesses with park-like features and extensive acreage.

With an enrollment PK-12 of 2,308, the district consists of 3 elementary schools, 1 intermediate school, 1 middle school, 2 magnet schools, and 2 high schools. The intermediate school is designed for students in grades 5-6. The population of 347 students includes 39.5% eligible for free/reduced price meals. This percentage exceeds slightly that of the district (37.9%) and is in contrast to the state data of 26.9%. Almost all of the students are from homes in which English is the first language. Table 1 provides the District Profile for School 1 with comparison data for the district and state.

Table 1

University of Connecticut Sites: District, School 1, and State Profiles

	District	School	State
% of Students Eligible for Free/Reduced-Price Meals	37.9	39.5	26.9
% of Students with Non-English Home Language	1.5	2.0	12.6

Source: Connecticut State Department of Education, School and District Profiles (2005-2006)

The student population in the district is predominately African American (85.8%), with 6.5% Hispanic and 6.4% White. At the school level, 89.2% of the students are African American, with 6.9% Hispanic, and 3.5% White. Table 2 provides all of the District and School 1 Student Population Data.

Table 2

University of Connecticut Sites: District and School 1 Student Population Data

Race/Ethnicity	District Number	District Percent	School Number	School Percent
American Indian	2	0.1	0	0.0
Asian American	28	1.2	1	0.3
African American	1,979	85.8	310	89.2
Hispanic	151	6.5	24	6.9
White	147	6.4	12	3.5
Total	2,307			

Source: Connecticut State Department of Education, School and District Profiles (2005-2006)

District and School 2 Profile

School 2 is approximately 15 miles from a major city with the 2000 population recorded at 17,328 and a per capita income of $23,257. Of the adult population, 84% have earned a high school diploma, which is similar to the statistic of the previously described district. Large and small businesses are located throughout the town. Neighborhoods of homes are from different time periods. Substantial homes with multi-bedroom designs and 3-car garages are being built on former farmland.

The current school district enrollment for PK-12 is 2,640. The school that participated in the Algebra pilot research study is for students in grades 6-8, with an enrollment of 686 students. Of these students, 21.1% are eligible for free/reduced price meals, which is higher than the district's 17.3% and lower than the state figure of 26.9%.

Less than 10% of the students are from homes in which English is not the dominant language (7.7%), while 8.3% of the district's students and 12.6% of the state's students are from non-English speaking families, as indicated in Table 3.

Table 3

University of Connecticut Sites: District, School 2, and State Profiles

	District	School	State
% of Students Eligible for Free/Reduced-Price Meals	17.3	21.1	26.9
% of Students with Non-English Home Language	8.3	7.7	12.6

Source: Connecticut State Department of Education, School and District Profiles (2005-2006)

The school population is 84.5% White, 7.9% African American, and 5.5% Hispanic. These data are similar to the population data for the district, which includes 85.6% White, 6.4% African American, and 5.9% Hispanic students. Table 4 portrays the students' district and school profiles.

Table 4

University of Connecticut Sites: District and School 2 Student Population Data

Race/Ethnicity	District Number	District Percent	School Number	School Percent
American Indian	3	0.1	1	0.1
Asian American	51	1.9	13	1.9
African American	170	6.4	54	7.9
Hispanic	155	5.9	38	5.5
White	2,261	85.6	580	84.5
Total	2,640			

Source: Connecticut State Department of Education, School and District Profiles (2005-2006)

Yale University Schools

Yale University implemented the pilot research study in 2 districts, with 2 schools in one district and one school in the other district. Overviews of the demographic characteristics of the districts and schools are provided below.

District and Schools 3 & 4 Profiles

Schools 3 and 4 were part of the same district. According to the 2000 census, the district for School 3 and School 4 has a population of 43,026 with a per capita income of $25,947. Of the adult population, 85.7% have earned a high school diploma.

With an enrollment of PK-12 of 7,036, the district consists of 8 elementary schools, 2 middle schools, and 2 high schools. The district includes only 5.3% of students eligible for reduced price meals. This percentage is dramatically lower than that of the state (26.9%). The majority of students are from homes in which English is the first language. Table 5 provides the District Profiles for School 3 and School 4 with comparison data for district and state.

Table 5

Yale University Sites: District, School 3, School 4, and State Profiles

	District	School 3	School 4	State
% of Students Eligible for Free/Reduced-Price Meal	5.3	8.2	4.8	26.9
% of Students with Non-English Home Language	7.3	9.6	4.8	12.6

Source: Connecticut State Department of Education, School and District Profiles (2005-2006)

The student population in the district is predominately White (84.8%), with 8.8% Hispanic, 2.5% African American, and 3.7% Asian American. The distribution is similar in both schools. Table 6 provides all of the District, School 3, and School 4 Student Population Data.

Table 6

Yale University Sites: District and School 3 & School 4 Student Population Data

Race/Ethnicity	District Number	District Percent	School 3 Number	School 4 Percent	School 4 Number	School 4 Percent
American Indian	18	0.3	0	0	0	0
Asian American	257	3.7	23	2.9	31	3.6
African American	177	2.5	19	2.4	22	2.5
Hispanic	617	8.8	76	9.6	56	6.4
White	5,967	84.8	676	85.1	762	87.5
Total	7,036					

Source: Connecticut State Department of Education, School and District Profiles (2005-2006)

District and School 5 Profile

According to the 2000 census, the district for School 5 has a population of 23,035 with a per capita income of $29,919. Of the adult population, 85.7% have earned a high school diploma.

The current school district enrollment for PK-12 is 3,925. The school that participated in the Algebra pilot research study is for students in grades 6-8, with an enrollment of 968 students. Of these students, 9.0% are eligible for free/reduced price meals, which is higher than the district's 6.9% and lower than the state figure of 26.9%. Less than 10% of the students are from homes in which English is not the dominant language (6.9%), while 6.4% of the district's students and 12.6% of the state's students are from non-English speaking families, as indicated in Table 7.

Table 7

Yale University Sites: District, School 5, and State Profiles

	District	School 5	State
% of Students Eligible for Free/Reduced-Price Meal	6.9	9.0	26.9
% of Students with Non-English Home Language	6.4	6.9	12.6

Source: Connecticut State Department of Education, School and District Profiles (2005-2006)

The student population is 84.6% White, 7.7% Asian American, 3.9% African American, and 3.6% Hispanic. This distribution is similar to the population data for the district, which includes 85.5% White, 6.9% Asian American, 4.0% African American, and 3.4% Hispanic. Table 8 portrays the students' district and school profiles.

Table 8

Yale University Sites: District and School 5 Student Population Data

Race/Ethnicity	District Number	District Percent	School 5 Number	School 5 Percent
American Indian	6	0.2	1	0.1
Asian American	272	6.9	75	7.7
African American	158	4.0	38	3.9
Hispanic	132	3.4	35	3.6
White	3,357	85.5	819	84.6
Total	3,925			

Source: Connecticut State Department of Education, School and District Profiles (2005-2006)

Part IV: Connected Mathematics Program

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University of Connecticut

The *Connected Mathematics Program* (CMP) was the product of a project funded by the National Science Foundation between 1991 and 1996. CMP is a complete mathematics program where students develop mathematic concepts, skills, and processes by participating in collaborative problem-based learning experiences. The overarching goal of the CMP is for students to be able to reason and communicate proficiently in mathematics (*Connected Mathematics*, 2006a). For students to attain this goal, they should acquire knowledge of and skill in the use of the vocabulary, forms of representation, and intellectual methods of the discipline of mathematics (*Connected Mathematics,* 2006a).

To foster mathematical reasoning and communication, the CMP was based on a three-phase spiral curriculum, which exposes students to mathematical concepts that are embedded in a narrative context. Students complete an initial problem based upon a story narrative during the *Application* phase, which requires the application of newly acquired mathematical knowledge and skills to solve the problem. The *Connection* phase offers students the opportunity to make conceptual connections between newly learned knowledge about a mathematical area (e.g., Algebra) and other mathematical areas (e.g., measurement, probability). The *Connection* phase requires students to complete problems that usually draw on two or more mathematical areas, which permits students to develop an integrated view and knowledge of the discipline. The final phase, *Extensions*, provides students with the opportunity to solve more complex problems about the new mathematical area. The problem-based curriculum design also provides teachers with the opportunity to teach mathematical skills and processes in context, including: representing, reasoning, comparing, measuring, estimating, modeling, connecting, using tools and becoming mathematicians.

The successful implementation of the CMP requires teachers to adopt a facilitation style of teaching. While teachers will utilize direct instruction to teach mathematical concepts during the program, the collaborative problem-based curriculum requires teachers to act as mathematical role models during instruction, and to provide on-the-spot teaching where necessary to individuals and groups of students. This teaching approach is based upon the notion that the circumstances under which students learn affect what they learn (*Connected Mathematics*, 2006a). Through a participatory instructional approach, teachers using the CMP may assist students to develop an integrated view of the discipline, specifically the application of mathematical concepts and skills to reason and communicate mathematically. This approach contrasts significantly with more traditional approaches that see students learn algebraic rules and processes out of context, with no relation to other mathematical areas, and in isolation from their peers. There is little evidence that students learn algebraic reasoning from memorizing rules and symbols in an isolated fashion (Lappan, 2004).

Numerous research studies conducted to evaluate the effectiveness of the CMP have indicated that it is an effective middle school curriculum (*Connected Mathematics,* 2006b). The results of the research studies (*Connected Mathematics*, 2006b) have consistently shown that:

- CMP students do as well as, or better than, non-CMP students on tests of basic skills;
- CMP students outperform non-CMP students on tests of problem-solving ability;
- CMP students can use basic skills to solve important mathematical problems and are able to communicate their reasoning; and
- By the end of grade 8, CMP students show a considerable ability to solve non-routine Algebra problems and demonstrate a strong understanding of linear functions.

Research studies examining the effectiveness of the program for students from special populations (including minority, gifted, and low SES students) showed that CMP students from such populations showed greater gains than their non-CMP counterparts.

Part V: Descriptions of Instruments

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University of Connecticut

After a thorough review of existing instruments, we adopted several instruments responsive to the research questions. We chose out-of-level achievement assessments to ensure that there would be an opportunity for students to demonstrate their growth in achievement on the posttests.

Achievement Measures

We selected the Iowa Tests of Basic Skills (grade 8) and the Iowa Algebra Aptitude Test (grade 8) to screen and identify potential students for their participation in the Algebra research study. The *Connected Mathematics 2* Unit Test (grade 7) was used to assess the level of pre/post mastery of the curriculum focusing on algebraic understanding. The following are brief descriptions of these three achievement measures:

Iowa Tests of Basic Skills: Form A (H. D. Hoover, S. B. Dunbar, & D. A. Frisbie) (2001)
The Iowa Tests of Basic Skills (ITBS) provide a comprehensive assessment of student progress in the basic skills. They consist of a Complete Battery (reading, language arts, mathematics, social studies, and science), a Core Battery (reading, language, and mathematics), and a Survey Battery (shortened version of Core Battery). All new test content is aligned with the most current content standards, curriculum frameworks, and instructional materials. The test was standardized on a national sample of students K-9, with approximately 3,000 students per level per form completing the tests. Internal consistency estimates using KR 20 varied between .79 and .98. Students in the standardization sample represented various types of communities, ethnicity, race, and socioeconomic status. The standardization sample included public, parochial, and non-parochial school. Schools in the standardization were further stratified by socioeconomic status. Data from these sources were used to develop special norms for a variety of groups (e.g., race/ethnicity, public school).

The Iowa Tests of Basic Skills offer multiple subtests in mathematics. The Mathematics Problem Solving and Data Interpretation subtest for grade 8 students was selected to assess students' knowledge and skills.

Iowa Algebra Aptitude Test: Forms 1 and 2 (4th ed.) (H. L. Schoen, & T. N. Ansley) (1993)
The Iowa Algebra Aptitude Test (IAAT), developed out of the Iowa Testing Program, consists of four subtests (Interpreting Mathematical Information, Translating to Symbols, Finding Relationships, and Using Symbols and is used with grade 8 students middle school students.

Reliability estimates were obtained using KR20s and range from .67 to .84 and total test reliability over .90. The strength of the test is in the careful adherence to the NCTM Standards (1989) in the development of the IAAT. The procedure for development included complete review of current texts and the mathematics educational research literature, careful item review and field-testing, and content review by mathematics educators. Criterion-related validity studies reveals that the IAAT does a good job of predicting 9th grade Algebra grades and test scores. IAAT scores were also significantly related to ITBS Mathematics Total scores (r = .69) and ITED Quantitative Thinking scores (r = .48). Multiple regression analyses demonstrate "that the IAAT composite scores did indeed significantly add to the prediction of success in Algebra 1." (The University of Iowa, 2006, p. 14).

Connected Mathematics 2 Unit Test, Variables and Patterns (G. Lappan) (2006)
Connected Mathematics 2, Variables and Patterns unit is designed for grade 7 students. After students were selected for participation in the pilot study, teachers administered the *Connected Mathematics* unit test as a pretest prior to starting the work with Investigations 1-4 comprising the selected unit. The *Connected Mathematics* unit test consists of 12 items, of which a few items were coded as "answers will vary." Given the limited number of items and item types, three algebraic equations were added to raise the potential ceiling on the test used on a pre/post basis.

Attitudes Toward Mathematics (M. Tapia) (2004)
Dr. Martha Tapia from Berry College, GA developed the Attitudes Toward Mathematics Inventory, which reflects research-based evidence in the following categories: confidence, anxiety, value, enjoyment, motivation, and parent/teacher expectations. Originally, 49 items were created and subjected to a factor analysis with 545 students, of which 540 were in high school and 5 were in grade 8. The response scale was: (1) *strongly disagree*, (2*) disagree*, (3) *neutral*, (4) *agree*, and (5) *strongly agree*. Nine items were removed from the item set, resulting in an alpha reliability value of .97. The score for the instrument was the sum of all ratings, with a mean of 137.36 and standard deviation of 28.93, and a standard error of measurement of 5.28. Four factors were identified, accounting for 55% of the variance. Factors identified were self-confidence, value, enjoyment, and motivation. As the information about the items assigned to each factor and the identification of items to be reversed scored were not available in the Tapia and Marsh (2004), the author provided the required information.

The Pearson correlation coefficient determined the test-retest reliability after 4 months with 64 students. The correlation coefficient for the total scale was .89, and the subscales were: self-confidence (.88); value (.70), enjoyment (.84), and motivation (.78). The total scores and subscales scores were stable.

Instrument Development

The pilot study of teaching Algebra to grade 6 students for a total of 30 hours required the development of several instruments to document and trace all phases of implementation.

Teachers' Logs

The University of Connecticut research team developed a Teacher's Log to track the progress of each teacher's implementation of the lessons on a weekly basis (see Appendix A). The logs were based on the components of lesson design as interpreted by *Connected Mathematics 2*. We asked teachers to list the Investigation number and then "Briefly Describe Your Approach to the Investigation." The CMP divides the content of *Variables and Patterns* unit into four investigations (see program description above). Next they described how they introduced and implemented the lesson under the following categories: Launch, Explore, and Summarize. The teachers who were using technology during the Investigations 1-3 commented on their use of graphing calculators and Excel. The next section of the log asked teachers to: "Describe the Students' Reactions to This Investigation." The final request was to "List Students Who Completed ACEs," which are Applications, Connections, and Extensions at the end of each investigation to reinforce and enhance learned skills and concepts.

Teacher Interview Questions

Teacher Interview Questions (Oliver, 2006a) and the Interview Questionnaire for Principals (Oliver, 2006b) were developed from a thorough review of extant literature (see Appendix B). The factors explored by the interview protocols included:

A. Beliefs/self-efficacy about own math abilities (particularly Algebra);
B. Personal epistemology regarding mathematic instruction (problem solving, constructivist approach, collaborative learning, etc);
C. Instructional efficacy (how confident the teachers feel in teaching the subject matter—mathematics in general, and specifically *Connected Mathematics*); and
D. Beliefs about high potential math students (characteristics, instructional needs, etc).

The research team posed 12 questions to participating teachers asking them to reflect on initial training, impact of experience on teaching, and beliefs about screening, identifying, and teaching high potential math students. Teachers were also asked to comment on the best methods for developing the talents of high potential math students, and to describe their level of efficacy in working with high potential math students.

Interview Questionnaire for Principals

The University of Connecticut research team also created a brief set of interview questions to provide some broad-based information about teaching and learning. The Interview Questionnaire for Principals (Oliver, 2006b) was designed to explore the impact of teacher factors on the effective instruction of high potential math students (see Appendix C).

The 7-item questionnaire posed open-ended questions such as: What do you consider to be the most important components of good math instruction? Are high potential math students easy to instruct?

Math Teacher Questionnaire

The Yale University research team created the Math Teacher Questionnaire (see Appendix D). They based items on prior assessment tools used in large-scale and small-scale research studies. Participating teachers were asked to reflect on their Algebra class as they responded to a series of 9 items with several sub-items to be completed using different response sets. Item 1 (sub-items a-i) asked teachers to indicate the percentage of time spent on specific activities, yielding a total of 100%. Sub-items included: reviewing assigned seatwork, working problems with your guidance, listening to you re-teach and clarify content/procedures. Item 2 focused on amount of time devoted to seatwork, which was followed by Item 3 with 4 sub-items related to details concerning the seatwork with a frequency response scale of (1) *never*, (2) *rarely,* (3) *sometimes*, (4) *often*, (5) *always*.

Item 4 asked teachers to respond to the same frequency response scale on 9 sub-items related to mathematics skills or tasks, for example: work on fractions and decimals; interpret tables, charts, or graphs; explain their answers.

The same frequency response scale used above was selected for Items 5-7. Item 5 addressed possible limitations to teaching with 4 sub-items such as: students with different academic abilities; uninterested students. Items 6 and 7 and their sub-items referred to use of calculators and computers.

Items 8 and 9 checked the extent of agreement or disagreement with sub-items on technology (Item 8) and students' attitudes toward math (Item 9). Response scale was (1) *strongly disagree*, (2) *disagree*, (3) *neutral*, (4) *agree*, (5) *strongly agree*.

Classroom Observation Scale

The Classroom Observation Scale (De Wet & Gubbins, 2006) consists of 14 close-ended items with a 4-point response scale: (1) *not effective*, (2) *partially effective*, (3) *moderately effective*, and (4) *very effective* (see Appendix E). The first set of items (1-4) focuses on objectives; lessons and assignments; prior knowledge, skills, and understandings; and reasoning skills. Items 5-8 highlight student engagement; reactions to lessons; questioning techniques, and communication. The final set of items (9-14) emphasizes engagement of students' intellect, assessment of understanding, discourse, knowledge and understanding, and disposition toward mathematics.

Student Questionnaire

The University of Virginia research team developed the Mathematics Classroom Practices Survey: Algebra Research Study (see Appendix F). Students were asked to reflect on their experiences in the Algebra research study. The 8-item survey includes 6 closed-ended items and 2 open-ended items. Sample items include:

1. If a friend asked you about this math program, what 3 words would you use to describe the program?
2. Describe the activity or activities that you did in this class that helped you learn the most math.

Part VI: Description of the Intervention and Results

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University of Connecticut Research Study: Schools 1 and 2

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Yale University Research Study: Schools 3, 4, and 5

Algebra I is a pre-requisite course for taking more advanced level mathematics courses (e.g., Algebra II, Geometry, Trigonometry, Calculus) and science courses (e.g., Chemistry, Physics) in high school. Aiding students in visualizing spatial relationships between and among variables may improve student success in Algebra, thereby equalizing opportunity for enrollment in more advanced level mathematics and science classes. We identified grade 6 students who performed well on the Iowa Tests of Basic Skills Mathematical Problem Solving and Data Interpretation subtest (grade 8) and the Iowa Algebra Aptitude Test (grade 8), and who earned at least a B in math. Students were assigned to classrooms with or without the use of technology (graphing calculators and Excel) while they were completing the *Connected Mathematics Program, Variables and Patterns* unit's Investigations 1-3. All students used graphing calculators and Excel with Investigation 4.

Research Questions

To investigate the impact of the after-school research study emphasizing algebraic understanding, the following modified research questions were posed to analyze the data for the University of Connecticut research sites only:

1. Does involvement in an after-school Algebra pilot research study impact students' mathematics achievement?
2. Does participation in the mathematics intervention affect student self-efficacy, and positive attitudes and interest in mathematics?
3. What are students' perceptions of the mathematics classroom practices in the after-school pilot research study?
4. What are teachers and administrators' perceptions of teaching and learning mathematics?

Sample

University of Connecticut

Of the 110 students screened for the University of Connecticut Algebra research study, 73 participated (School 1, n = 30; School 2, n = 43). Table 9 presents the demographic characteristics of the participants. Males outnumbered the females in both schools. Overall, 44 males and 29 females were involved. The majority of students from School 1 were African Americans and the majority in School 2 was White.

Table 9

University of Connecticut Sites: Demographic Characteristics of Participants (N = 73)

	School 1		School 2		Total
Gender	Number	Percent	Number	Percent	Number
Male	17	56	27	63	44
Female	13	43	16	37	29
Total	30	100*	43	100	73
Ethnicity					
Asian American	0	0	1	2	1
African American	24	80	1	2	25
Hispanic/ Latino	1	3	1	2	2
White	2	7	38	88	40
Other	3	10	2	5	5
Total	30	100	43	100*	73

*rounded to 100%

School 1 students were divided into 2 groups and worked with 1 teacher who used technology or 1 teacher who did not use technology during the Investigations 1-3. School 2 students were divided into 3 groups, with 2 groups working with teachers using technology and 1 group without access to technology. At the end of CMP, Investigation 3, all students completed the posttests for the research study: Iowa Tests of Basic Skills, Mathematical Problem Solving and Data Interpretation subtest and Iowa Algebra Aptitude Test. Then all students accessed technology for Investigation 4. (Note: Students completed the CMP unit test prior to Investigation 1 and at the end of Investigation 4.)

Five teachers participated in the pilot research study. Three taught math in middle schools for their current assignments, one teacher taught math in high school, and one teacher was a long-term substitute in the process of seeking certification as a middle school teacher. Prior to becoming educators, 4 of the 5 teachers worked in professions focusing on mathematics (insurance, accounting). The teachers were not familiar with the CMP. The University of Connecticut research team provided 2 days of professional development focusing on the *Connected Mathematics* approach to teaching Algebra and familiarizing them with the graphing calculators and Excel. The first day of professional development convened prior to the research study and the second day occurred after the teachers had several weeks of experience. Throughout the implementation of the research studies, research team members communicated via emails and phone calls. The team also was on site conducting observations and was available to respond to any questions.

Yale University

Overall, 90 students participated in the Yale University Algebra research study (School 3, n = 32, School 4, n = 31, School 5, n = 27). Table 10 presents the demographic characteristics of the participants. Males outnumbered females in all schools. Overall, 56 males and 34 females were involved. The majority of students from School 3 and School 4 were White, while a fairly even number of students from School 5 were White or Asian American.

Table 10

Yale University Sites: Demographic Characteristics of Participants (N = 90)

	School 3		School 4		School 5		Total
Gender	Number	Percent	Number	Percent	Number	Percent	Number
Male	22	69	19	61	15	56	56
Female	10	31	12	39	12	44	34
Total	32	100	31	100	27	100	90
Ethnicity							
Asian American	2	6	1	3	10	37	13
African American	1	3	0	0	0	0	1
Hispanic/Latino	1	3	0	0	0	0	1
White	27	84	28	90	13	48	68
Other	0	0	0	0	2	7	2
Unknown	1	3	2	7	2	7	5
Total	32	100*	31	100	27	100*	90

*rounded to 100%

Students were divided into 2 groups at each school. Each group worked with 1 teacher who used technology or 1 teacher who did not use technology. Overall, 6 teachers were involved in the study.

Research Question 1: Achievement Results

Research Question 1 focused on the potential impact of technology on the mathematics achievement of students involved in an after-school Algebra research study.

University of Connecticut Achievement Results

To provide the University of Connecticut sites with a summary of the impact of the Algebra study, data were analyzed across and by Schools 1 and 2. Given the small sample size, the University of Connecticut data could not be analyzed by Intervention 1 (technology) and Intervention 2 (no technology). The following revision guided the achievement research question for the University of Connecticut sites is:

1. Does involvement in an after-school Algebra research study impact students' mathematics achievement?

Achievement Differences Across Schools and Across Interventions

To compare the pre/posttest differences across schools and across interventions, paired samples *t* tests were conducted on each matched set of achievement data: ITBS pre/post; IAAT pre/post; and CMP pre/post (12 items); CMP pre/post (15 items). The mean differences for each set of data indicated statistically significant increases from pre to posttests. The paired samples *t* test results follow (see Table 11). There are different viewpoints concerning the calculation of effect sizes (ES) when using paired samples *t* tests. Some researchers advocate one formula over another. Cohen's *d* was used to calculate effect sizes in Table 11. According to Cohen (1988), the parameters are as follows: 0.2 = small effect; 0.5 = medium effect; >0.8 = large effect. The effect sizes in Table 11 indicate that the results of pre/post achievement tests yielded medium effect sizes for the ITBS and IAAT and large effect sizes for the CMP 12 and CMP 15.

Table 11

University of Connecticut Sites: Paired Samples *t* Tests for Achievement Tests

	Pre		Post				
Assessment	*M*	*SD*	*M*	*SD*	*df*	*t*	*ES*
ITBS SS	255.37	29.71	272.71	28.03	72	6.903*	0.600
IAAT SS	152.39	6.42	157.48	8.75	70	6.476*	0.663
CMP 12 Raw	5.13	2.19	7.82	2.61	66	9.257*	1.116
CMP 15 Raw	5.72	2.25	8.49	2.70	66	9.143*	1.114

*$p < .001$.

School 1 Achievement Results Across Interventions

For the School 1 paired samples *t* tests, the mean differences indicated increases in achievement on the ITBS, IAAT, CMP (12 items), and CMP (15 items) pre and posttests. The *t* tests for paired samples yielded the results indicated in Table 12, establishing statistically significant differences between the pre and posttests on all achievement measures. The effect sizes for the ITBS and IAAT were medium, while the effect sizes for the CMP 12 and CMP 15 were considered large or greater than 0.8.

Table 12

University of Connecticut School 1: Paired Samples *t* Tests for Achievement Tests

	Pre		Post				
Assessment	*M*	*SD*	*M*	*SD*	*df*	*t*	*ES*
ITBS SS	240.57	30.60	260.27	31.39	29	-4.48*	0.635
IAAT SS	150.27	5.94	154.93	7.78	29	-4.49*	0.673
CMP 12 Raw	4.79	1.83	7.14	2.64	27	-5.02*	1.034
CMP 15 Raw	5.14	1.78	7.75	2.77	27	-5.19*	1.121

*$p < .001$.

School 2 Achievement Results Across Interventions

Paired samples *t* tests were used to determine pre and posttest differences for the achievement tests for School 2. There were statistically significant differences from pre to posttests on the achievement tests, as indicated below (see Table 13). As with School 1, the effect sizes for School 2 on the ITBS and IAAT are medium, while the effect sizes for the CMP 12 and CMP 15 were large or greater than 0.8.

Table 13

University of Connecticut School 2: Paired Samples *t* Tests for Achievement Tests

	Pre		Post				
Assessment	*M*	*SD*	*M*	*SD*	*df*	*t*	*ES*
ITBS SS	265.70	24.51	281.37	21.89	42	-5.27*	0.674
IAAT SS	153.95	6.37	159.34	9.02	40	-4.75*	0.690
CMP 12 Raw	5.38	2.41	8.31	2.50	38	-7.94*	1.193
CMP 15 Raw	6.13	2.47	9.03	2.55	38	-7.60*	1.155

*$p < .001$.

Students involved in the Algebra research study improved their mathematics achievement. The mastery of the content related to algebraic understanding was evident as students' became more adept in creating and interpreting tables, graphs, and equations.

Yale University Achievement Results

Research Question 1 focused on the potential impact of technology on the mathematics achievement of students involved in an after-school Algebra research study. To compare the pre and posttest differences across schools and across interventions, paired samples *t* tests were conducted on each matched set of achievement data: ITBS pre/post; IAAT pre/post; and CMP pre/post (12 items); CMP pre/post (15 items). For the overall population examined, ITBS scores dropped slightly from pre to posttest, while the IAAT and CMP (12 and 15 items) scores all increased. Table 14 displays the mean paired samples *t* test results for each type of assessment.

Table 14

Yale University Site: Paired Samples *t* Tests for Achievement Tests

	Pre		Post			
Assessment	*M*	*SD*	*M*	*SD*	*df*	*t*
ITBS SS	299.09	15.74	298.35	23.54	78	.303
IAAT SS	162.99	11.13	168.77	10.75	78	-5.573*
CMP 12 Raw	6.08	2.78	8.71	2.16	79	-7.979*
CMP 15 Raw	6.60	2.96	9.56	2.39	79	-8.471*

*$p < .001$.

In addition to examining the overall scores, analyses of the results across treatment type were performed. The mean difference indicated increases in IAAT and CMP (12 and 15), while the ITBS scores remained approximately the same, in both the technology (treatment) and no additional technology (control) cases. The gains were not significantly different in either treatment case. Table 15 shows the specific changes for the control and treatment conditions, generated through paired samples *t* tests.

Table 15

Yale University Site: Paired Samples *t* Tests by Treatment for Achievement Tests

	Pre		Post			
Assessment	*M*	*SD*	*M*	*SD*	*df*	*t*
Control						
ITBS SS	297.55	15.77	297.34	22.35	37	.063
IAAT SS	161.60	9.60	167.93	8.28	39	-4.753
CMP 12 Raw	6.41	2.75	9.41	1.65	38	-6.927
CMP 15 Raw	6.95	2.96	10.15	1.80	38	-6.728
Treatment						
ITBS SS	300.51	15.79	299.29	24.82	40	.347
IAAT SS	164.41	12.48	169.64	12.86	38	-3.244
CMP 12 Raw	5.76	2.81	8.05	2.39	40	-4.628
CMP 15 Raw	6.27	2.96	9.00	2.74	40	-5.322

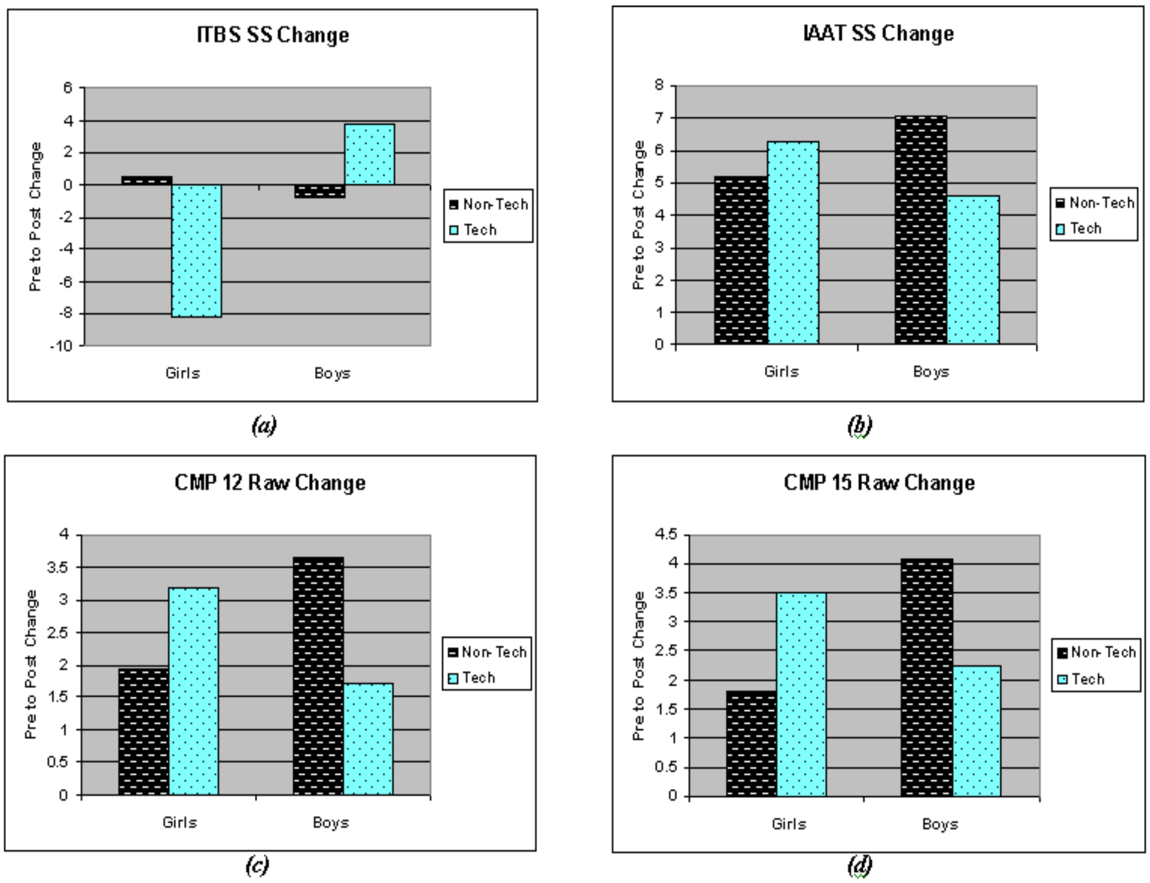
Further analyses resulted in paired samples *t* tests being performed where treatment type and gender were both considered. The changes in the mean IAAT, CMP

12, and CMP 15 pre and post scores were different for female and male participants in the two conditions. The presence of technology (treatment) increased the mean female scores more than the mean male scores, while the no additional technology condition (control) resulted in a greater rise in mean male scores than mean female scores. The results for the ITBS assessment were different than the other three assessments. The mean ITBS scores of females in the technology condition and males in the control condition actually decreased. Table 16 and Figure 1 summarize the paired samples *t* test results numerically and graphically for each assessment, when gender and condition are both considered.

Table 16

Yale University Site: Male-Female and Control-Treatment Achievement Tests via Paired Samples *t* Tests

	Pre		Post			
	M	*SD*	*M*	*SD*	*df*	*t*
Female, Control						
ITBS SS	300.38	15.70	300.94	21.37	15	-.11
IAAT SS	162.94	9.14	168.13	6.89	15	-2.74
CMP 12 Raw	7.60	2.77	9.53	1.55	14	-2.66
CMP 15 Raw	8.33	3.20	10.13	1.41	14	-2.42
Female, Treatment						
ITBS SS	299.24	14.32	291.00	31.27	16	1.11
IAAT SS	162.93	9.97	169.20	13.69	14	-2.78
CMP 12 Raw	4.81	3.17	8.00	2.68	15	-4.03
CMP 15 Raw	5.25	3.19	8.75	3.07	15	-4.55
Male, Control						
ITBS SS	295.50	15.85	294.73	23.18	21	.17
IAAT SS	160.71	9.98	167.79	9.24	23	-3.85
CMP 12 Raw	5.67	2.51	9.33	1.74	23	-7.30
CMP 15 Raw	6.08	2.48	10.17	2.04	23	-7.32
Male, Treatment						
ITBS SS	301.42	16.99	305.17	17.46	23	-1.43
IAAT SS	165.33	13.94	169.92	12.61	23	-2.05
CMP 12 Raw	6.36	2.43	8.08	2.24	24	-2.77
CMP 15 Raw	6.92	2.66	9.16	2.56	24	-3.31



<u>Figure 1.</u> Difference between pre and posttest scores by gender and condition for (a) ITBS, (b) IAAT, (c) CMP12, and (d) CMP15.

Research Question 2: Attitudes Toward Mathematics

University of Connecticut

To determine if involvement with the after-school pilot research study affected students' self-efficacy, and positive attitudes and interest in mathematics, students completed the Attitudes Toward Mathematics survey on a pre and posttest basis. Tapia first used this instrument with high school students and identified four factors: self-confidence, value, enjoyment, and motivation. The score for the instrument was the sum of all ratings, with a mean of 137.36 and standard deviation of 28.93.

Given the small sample size for the University of Connecticut sites, data were analyzed across the four factors of the instrument to answer Research Question 2, focusing on change in self-efficacy, and positive attitudes and interest in mathematics. The mean pretest score for students participating in the Algebra research study was 161.83, and the posttest score was 166.92. Table 17 presents the results of paired samples *t* tests, indicating there was no significant difference between the pre and posttest results across schools.

Table 17

University of Connecticut Site: Paired Samples *t* Tests for Attitudes Toward Mathematics

	Pre		Post			
Assessment	*M*	*SD*	*M*	*SD*	*df*	*t*
Math Attitude	161.83	28.24	166.92	19.25	64	-1.455

Data also were analyzed by School 1 and School 2. There were no significant differences between pre and posttest results. The pretest mean for School 1 (n = 29 students) was 163.86 (*SD* = 27.71) and the posttest mean was 169.17 (*SD* = 16.47). The School 2 (n = 36) pretest mean was 160.19 (*SD* = 28.95) and the posttest mean was 165.11 (*SD* = 21.28). The *t*-tests analyses of paired samples yielded the following: (School 1 (t =-1.145, df = 28, p>.05); School 2 (t =-.954, df = 35, p>.05)). Participation in the Algebra research study did not affect students' self-efficacy, and positive attitude and interest in mathematics. Participating students' pretest mean (161.83, *SD* = 28.24) and posttest mean scores (166.92, *SD* = 19.25) were considerably higher than the mean scores of the high school students who participated in Tapia's instrument validation (*M* = 137.36).

Yale University

Students completed the Attitudes Toward Mathematics (Tapia) survey on a pre and posttest basis to help determine if involvement with after-school pilot research study affected students' self-efficacy and positive interest and attitudes in mathematics. The score for the instrument was the sum of all ratings. The mean pretest score for all students participating in the Yale Algebra research study was 139.89, and the mean posttest score was 141.33. Table 18 presents the results of paired samples *t* tests, indicating there were no significant differences between the pre and posttest results overall or by school.

Table 18

Yale University Site: Paired Samples *t* Tests for Attitudes Toward Mathematics

	Pre		Post			
Assessment	*M*	*SD*	*M*	*SD*	*df*	*t*
Math Attitude	139.89	10.16	141.33	11.50	75	-1.302
School 3	141.39	9.14	143.39	12.13	27	-1.289
School 4	139.54	12.28	141.29	12.91	23	-.777
School 5	138.50	9.09	138.96	8.97	23	-.226

The data were also analyzed by condition and gender. There were no significant changes between pre and posttest attitudes toward mathematics in either condition or gender. Table 19 presents the results of paired samples *t* tests.

Table 19

Yale University Site: Control, Treatment, and Gender Paired Samples *t* Tests for Attitudes Toward Mathematics

	Pre		Post			
Math Attitude	*M*	*SD*	*M*	*SD*	*df*	*t*
Control	138.33	10.65	141.69	11.73	38	-2.032
Treatment	141.54	9.47	140.95	11.39	36	.428
Female	136.34	11.06	137.45	12.52	28	-.662
Male	142.09	9.00	143.72	10.24	46	-1.116

The results that proved interesting occurred when analysis was performed across gender and condition together. Females in the control group showed a small decrease in math attitude, while male's math attitude increased by 6 points in the control group. Meanwhile, the mean math attitude score of females in the treatment group increased by nearly 3 points and males in the treatment group decreased by nearly 3 points. These pre and posttest math attitude scores are shown in Table 20. Figure 2 serves to graphically highlight the differences between the mean pre and posttest math attitude scores.

Table 20

Yale University Site: Control and Treatment With Gender Paired Samples *t* Tests for Attitudes Toward Mathematics

	Pre		Post			
Math Attitude	*M*	*SD*	*M*	*SD*	*df*	*t*
Female, Control	137.47	11.46	137.00	13.23	14	.185
Female, Treatment	135.14	10.91	137.93	12.19	13	-1.300
Male, Control	138.88	10.33	144.63	9.88	23	-2.784
Male, Treatment	145.43	5.88	142.78	10.73	22	1.552

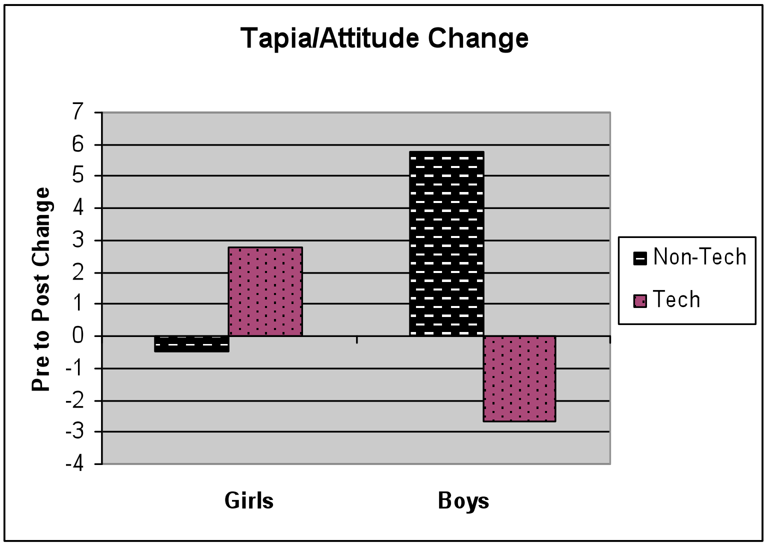


Figure 2. Difference between pre and posttest scores on the Attitude Towards Mathematics (Tapia) by gender and condition.

Research Question 3: Students' Perceptions of Mathematics Classroom Practices

University of Connecticut

In response to Research Question 3 (What are students' perceptions of the mathematics classroom practices in the after-school pilot research study?), students from Schools 1 and 2 were asked to fill out an 8-question survey at the conclusion of the research study. In total, 69 of the 73 students, yielding a 95% return rate, from the 2 schools completed the survey, including students who used technology and those who did not.

The content of the Student Questionnaire focused on reactions to the Algebra research study and the extent to which students found the program helpful, challenging, interesting, and different from their math classes. Students' reactions to the Algebra program are summarized by each question.

Responses to Question 1 (*If a friend asked you about this math program, what 3 words would you use to describe the program?*) can be roughly separated into 4 descriptive subcategories. Out of a possible 207 (3 x 69) words used to describe the experience, the following patterns were determined:

- 49% (102 words) can be classified as enthusiastically positive (e.g., fun, exciting, awesome);
- 32% (67 words) described the program as a form of beneficial learning (e.g., challenging, educational, worthwhile); and
- 9% (19 words) described the work as hard, difficult, confusing, long, or boring; while 3 word choices (easy, social, and "not hard") can be

> interpreted as indicators that the program was still not challenging for these individual students.

Looking at the words in isolation shows the majority of students had favorable impressions of the after-school program; however, the combinations of words used by some individual students offer an interesting picture, as well. It worth noting that students in this study mixed words such as "fun" (signifying enjoyment) with those describing learning as "challenging," and/or "hard," right along with "wonderful." A key finding was that learning in this context was, for many, enjoyable in many different ways, even though it consisted of 3 hours after school of hard math work. For example, the 4 students below wrote words that may appear, at first glance, to be unlikely partners. As noted by the raw scores on the ITBS, IAAT, and the *Connected Mathematics 2* unit test, 3 of these students listed below made substantial gains in scores (see Table 21).

Table 21

University of Connecticut Site: Analysis of Students' Description of the Algebra Program

Student #	Words Used	ITBS		IAAT		CMP	
		Pre	Post	Pre	Post	Pre	Post
		Raw Scores					
S45	Hard, challenging, fun	17	22	27	44	na	8
S13	Fun, not hard	17	17	30	36	4	10
S28	Long, easy, fun	13	12	26	29	4	9
S25	Fun, hard, wonderful	15	16	32	36	5	4

Activities related to graphs were the most common response to both Question 2 (*Describe the activity or activities that you did in this class that helped you learn the most math.*) and Question 3 (*Describe the activity or activities that you did in this class that were the least helpful to you in learning math.*). Forty-three percent (n = 30) students mentioned graphing as the most helpful activity (Question 2), yet graphing was included by 30% (n = 21) students as the least helpful activity (Question 3). Specific aspects of graphing were also noted as either "most helpful" or "least helpful." For example, S31 wrote that coordinate graphs (on paper) helped me learn the most math; however graphs on the calculator were least helpful. In contrast, for S23, graphs on paper were the least helpful. Distinctions between students with access to technology during Investigations 1-3 and those without access have not yet been made.

Question 4 asked "*How was this class different from your math classes at school?*" Of the students,

> 33% (n = 23) described the class as "harder" or "more challenging"
> 16% (n = 11) felt it was "faster-paced" or "more advanced"
> 16% (n = 11) of the students described it as "different"

10% of the students (n = 7) again used the word "fun" in response to this question
6% considered the food was a consideration
6% noted that the snacks made the classes different from their math classes during the school day.

Here are some representative comments followed by pre and post scores for specific students:

This class was different because you worked in groups a lot and there were activities that would help you learn (S89)

ITBS pre = 22, ITBS post = 27
IAAT pre = 34, IAAT post = 45
CMP pre = 6, CMP post = 12

It taught you harder things, and required you lots of thinking (S105)

ITBS pre = 17, ITBS post = 22
IAAT pre = 31, IAAT post = 23
CMP pre = 5, CMP post = 5

In this class we learn in the ways kids enjoy learning in (S66)

ITBS pre = 18, ITBS post = 25
IAAT pre = 30, IAAT post = 27
CMP pre = 5, CMP post = 6.

A small percentage compared the classes in a negative or neutral light. For example, one student (S57) wrote that the class was different because it was "much harder much more boring,"

ITBS pre = 24, ITBS post = 23
IAAT pre = 35, IAAT post = 36
CMP pre = 1, CMP post = 10

Two students (S13, S14) wrote, "It wasn't that different," yet both made substantial gains in 2 out of the 3 measures used.

ITBS pre = 17, ITBS post = 17
IAAT pre = 30, IAAT post = 36
CMP pre = 4, CMP post =10 (S13)

ITBS pre = 19, ITBS post = 18
IAAT pre = 33, IAAT post = 44
CMP pre = 5, CMP post =11 (S14).

For Question 5 (*What was most challenging about these math lessons?*), the responses were highly individual and specific, such as "learning the x and y axis" (S39), "understanding the vocabulary" (S28), "working by yourself" (S27), or "using fractions and turning them into decimals" (S97). The most frequent responses included working with the graphing calculator (n = 6) and dealing with equations (n = 9). Three students reported there was nothing challenging. Two responses are listed below with their raw scores:

I can't think of anything that was really challenging (S85)

ITBS pre = 20, ITBS post = 21
IAAT pre = 34, IAAT post = 43
CMP pre = 3, CMP post =10

nothing really (S109)

ITBS pre = 21, ITBS post = 28
IAAT pre = 51, IAAT post = 50
CMP pre = na, CMP post =12.

Question 6 then asked: *What was the least challenging about these lessons*? Nearly half (46%) of the students (n = 32) wrote that making graphs, charts, and or tables was the least challenging, while 6% (n = 4) noted that playing the games was not challenging.

From Question 7 (*How good are you at math? (a) = math whiz; (b) = very good; (c) average; (d) struggle; (e) I cannot do math well at all),* we learned that 69% (n = 48) rated themselves as "very good" at math. Twenty-four percent (n = 17) considered themselves "whizzes" at math, while 7% (n = 5) believed they were "average." None of the students in the study believed that they were below average.

Question 8 asked, "*How interesting were the math lessons?" (a) very interesting; (b) most were interesting; (c) only some were interesting; (d) some were not interesting; (e) most were not interesting*. Of the 67 responses to this question, 55% (*n* = 37) found most of the lessons were interesting and 26% (n = 18) of the students believed the lessons were "very interesting." While 14% (n = 10), believed only "some of the lessons were interesting," 1 student indicated "some were not interesting," and 3% (n = 2) marked most of the lessons as not interesting.

A look at the combination of Questions 7 and 8 (Table 22) shows that of the 24% of students (n = 17) who believed they were "whizzes" at math, 10% (n = 7) thought most of the lessons were very interesting, while only one "whiz" found most of the lessons "not very interesting."

Table 22

University of Connecticut Site: Students' Description of Perceived Math Ability and Interest Level of Project Lessons

Code for Q7/Q8	Description of Codes (Q7 + Q8)	Frequency
a/a	whiz *and* most lessons were very interesting * (see note #1)	7
a/b	whiz *and* most lessons were interesting	7
a/c	whiz *and* only some were interesting	1
a/d	whiz *and* some not very interesting * (see note #2 below)	1
a/e	whiz *and* most not very interesting * (see note #2 below)	1
b/a	I do very good math work *and* lessons very interesting	10
b/e	do very good math work *and* most not very interesting	1
c/a	I am average in math *and* lessons very interesting	1
b/b	do very good math work *and* most lessons were interesting	25
/c	only some lessons were interesting * (see note #3 below)	9
/d	only some lessons were not interesting * (see note #3 below)	1
/e	most lessons were not very interesting * (see note #3 below)	2

*Note #1: Scores for the 7 students who rated themselves as math whizzes and who found the lessons "very interesting" are listed in Table 23. These students (with the exception of S63, who showed a small decrease on the ITBS) made gains in each measure. In summary, students who rated themselves as math whizzes and who found the lessons "very interesting" consistently had scores above the group mean.

Table 23

University of Connecticut Site: Achievement Results for "Math Whiz" Students Who Rated Lessons as "Very Interesting"

	ITBS Pre	ITBS Post	IAAT Total Pre	IAAT Total Post	CMP Pre 12 Items	CMP Post 12 Items	CMP Pre 15 Items	CMP Post 15 Items
	Raw Scores							
Total Sample Mean	19	22	33	38	5	8	6	9
Student ID								
40	22	25	34	42	4	5	4	6
63	29	27	38	54	7	12	8	13
68	19	19	30	40	4	6	4	6
69	19	23	37	43	8	9	8	10
73	26	27	44	53	7	12	7	13
93	25	30	39	43	8	10	9	10
111	24	27	41	51	5	11	6	12
Mean score (7 students)	23	25	38	47	6	9	7	10

*Note #2: On the other hand, the 2 students who rated themselves as whizzes in math but found the lessons not very interesting, showed a decrease in most scores as shown in Table 24.

Table 24

University of Connecticut Site: Achievement for "Math Whiz" Students Who Rated Lessons as "Not Very Interesting"

	ITBS Pre	ITBS Post	IAAT Total Pre	IAAT Total Post	CMP Pre 12 Items	CMP Post 12 Items	CMP Pre 15 Items	CMP Post 15 Items
	Raw Scores							
Total Sample Mean	19	22	33	38	5	8	6	9
Student ID								
80	21	18	34	27	1	na	1	na
106	16	23	43	37	2	5	4	5

Student perceptions of how interesting they found the lessons appeared to positively influence student achievement. For example, one student (S2) rated himself/herself as "average" in math (by selecting (c) on Question 7), yet he found "most of the lessons interesting" (selecting (b) on Question 8). As noted below, this student made substantial gains:

S2: ITBS pre = 14, ITBS post = 24
IAAT pre = 29, IAAT post = 41
CMP pre = 5, CMP post = 7.

* Note #3: Students who found the lessons not as interesting, no matter how they reported their math skills, (those who rated the lessons as: (c) only some were interesting; (d) some were not interesting; or (e) most were not interesting) had scores below the mean (see Table 25).

Table 25

University of Connecticut Site: Scores for Students Who Rated Algebra Lessons as "Not Interesting"

	ITBS Pre	ITBS Post	IAAT Total Pre	IAAT Total Post	CMP Pre 12 items	CMP Post 12 Items	CMP Pre 15 Items	CMP Post 15 Items
	Raw Scores							
Total Sample Mean	19	22	33	38	5	8	6	9
Student ID								
6	18	13	28	28	3	4	5	6
38	16	18	36	44	7	7	7	8
11	16	23	29	39	5	7	5	7
48	12	18	27	42	5	9	6	9
14	19	18	33	44	5	11	5	12
59	24	25	38	43	4	9	5	10
107	18	27	28	40	6	6	7	7
66	18	25	30	27	5	6	5	6
105	17	22	31	23	5	5	5	5
106	16	23	43	37	2	5	4	5
57	24	23	35	36	1	10	2	11
80	21	18	34	27	1		1	
Mean score (12 Students)	18	21	33	36	4	7	5	8

Yale University

Students from the Yale University Algebra study were also asked to fill out an 8-question survey to gauge their perceptions of mathematics classroom practices in the after-school program. Only 33 of the 90 students completed the survey, yielding a 37% return rate. Responding students were part of both the technology and control classrooms.

Responses to Question 1 (*If a friend asked you about this math program, what 3 words would you use to describe the program?)* can be roughly separated into 4 descriptive subcategories. In line with the analysis approach above, of the 87 (3 x 33 with 12 blanks) words to describe the experience, the following patterns were determined:

- 39% (34 words) can be classified as enthusiastically positive (e.g., fun, exciting, interesting)
- 37% (32 words) described the program as a form of beneficial learning (e.g., rewarding, learning experience, educational)
- 20% (17 words) described the program negatively (e.g., boring, long, difficult)
- 5% (4 words) described the program as easy or not difficult

Only 3 students used solely negative words to describe the program. Some students juxtaposed negative and positive descriptors, such as long with interesting and educational. Several of the students who viewed a part of the program negatively, felt there was also something positive about the program.

Activities related to graphs were the most common response to both Question 2 *(Describe the activity of activities that you did in this class that helped you learn the most math)* and Question 3 *(Describe the activity of activities that you did in this class that were the least helpful to you in learning math)*. Forty-seven percent (n = 15) of the students responding mentioned graphing as the most helpful activity (Question 2), yet graphing was included by 32% (n = 8) as the least helpful activity (Question 3).

Students were also asked how they felt the program was different than their math classes (Question 4). Fifty-five percent (n = 17) of students responding found the class harder or more challenging than math classes at school. Other noted differences included the after-school program including seatwork, providing snacks, and being more fun. However, there was not consistent agreement among students on differences between the program and regular math classes, beyond finding the program more difficult than their math classes at school.

To determine some of the challenging pieces of the lessons, students were asked about the most (Question 5) and least (Question 6) challenging elements of the lessons. Graphs and equations were the most common lesson elements highlighted by students. The responses with the highest frequency for the most challenging element were equations (n = 9, 29%) and graphs (n = 5, 16%). The most common responses for least challenging element were graphs (n = 11, 39%) and tables (n = 5, 18%).

Students were also asked to rate how good they were at math (Question 7) and how interesting the math lessons were (Question 8). Forty-five percent of students (n = 15) considered themselves math whizzes and they all felt at least some of the lessons were interesting. Fifty-two percent (n = 17) of students considered themselves very good at math. The students who considered themselves very good at math expressed all levels of interest and disinterest in the math lessons. A single student self-perception was as average in math, but found most of the lessons interesting. Table 26 shows the juxtaposition of students' confidence and interest in math.

Table 26

Yale University Site: Students' Description of Perceived Math Ability and Interest Level of Project Lessons

Code for Q7/Q8	Description of Codes (Q7 + Q8)	Frequency
a/a	whiz *and* most lessons were very interesting * (see note #1)	4
a/b	whiz *and* most lessons were interesting	8
a/c	whiz *and* only some were interesting	3
b/a	I do very good math work *and* lessons very interesting	5
b/b	I do very good math work *and* most lessons were interesting	3
b/c	I do very good math work *and* only some were interesting	4
b/d	I do very good math work *and* some not very interesting	2
b/e	I do very good math work *and* most not very interesting	3
c/b	I am average in math *and* most lessons very interesting	1

Summary of Student Questionnaire Data

Taken as a whole, it appears the majority of students found the intensive after-school Algebra program fun, interesting, and exciting, even though it was in addition to (not a substitute for) their regular math classes. Many noted that the work differed from the regular classroom because the work was more difficult. Yet, the students in this study found hard, difficult, and challenging work in Algebra to be fun and exciting.

Student perception of interesting lessons was key. The few students who indicated they found "most lessons not very interesting" did not show similar gains as those who found the lessons "very interesting."

There were few commonalities in the list describing the most challenging activities within the classes, indicating a broad range of individual needs; however, as summed up by S80, "All of the investigations that we did helped a lot."

Summary of Teachers' Interviews

University of Connecticut

The teachers were interviewed midway through the intervention program to investigate their self-efficacy in using mathematics and teaching the discipline. Information was also collected to ascertain the teachers' knowledge concerning how to identify talented math students, and what instructional methods the teachers thought were most effective at developing the talent of high potential math students.

When asked to recall their own school experiences about learning Algebra, most interviewees identified having a poor teacher who taught from the book, used a skill and drill instructional approach, and did not use a variety of activities to teach mathematical concepts. One respondent stated that his "math teacher was mean, demanding, loud, and brash." Two respondents commented that their teachers were fun, enthusiastic, and appeared to have a passion for mathematics. When asked whether or not these experiences had an influence on their own teaching style, most teachers agreed that their school experiences, whether positive or negative, affected their instructional style.

Respondents who had a positive role model indicated that they tried to emulate the instructional style of their high school teachers. One respondent stated, "he realized the importance of being passionate in the classroom," and continued by explaining "you have to be excited as a teacher in order to excite your students." Another teacher commented that the enthusiasm from her high school teacher "permeated through her teaching." The respondent who had the "mean and brash" math teacher explained that he thought that "this was the way to teach math," and consequently taught in this manner until he participated in the intervention program. Two interviewees indicated that their high school math teachers did not influence their own instructional style. One of these respondents stated that she "taught differently because times had changed, and students were different," and continued to explain that students "wanted instant responses, had a short attention span, and liked to be entertained."

When the teachers were asked to gauge their own mathematic abilities, the 4 non-traditionally trained teachers asserted that they were "very confident" using math in everyday life activities. One respondent stated, "I never really think about how hard math is, or not being able to do it," and another commented that "it just comes naturally, I don't have to think about it at all." While the traditionally trained teacher was confident in her math abilities, she commented that her confidence had grown since she had taught math and that "learning to become a math teacher helped me to learn the tricks of math that I did not know before." Considering the high level of self-efficacy towards using math in daily activities, most of the respondents did not feel that they needed to improve their math skills. Two teachers did indicate however that they wished to stay current with topics that they perceived as being related to the discipline, most notably the field of technology.

The survey questions related to effective instructional practices for teaching mathematics generated a variety of responses. Consistent themes that were forwarded by all respondents included: the content knowledge of the teacher; awareness of strategies to cater for student differences; and the personal characteristics of the teacher. Most respondents mentioned the importance of developing expertise in content knowledge to be an effective teacher of mathematics, and qualified this assertion with statements such as "must possess a strong knowledge of content," and "have a thorough knowledge of the discipline." One respondent noted that, "a teacher should have a love for the subject" to be an effective teacher of mathematics. All teachers asserted that having an awareness of different learning styles and knowledge of how to cater to individual differences was an essential component of quality instruction. One teacher commented that it was important to be able to "use different techniques with different students," and another declared that teachers "should use a variety of ways to teach the subject." Most of the responses generated by questions about effective instructional practices were related to personal qualities of the teacher. To be an effective teacher, the respondents believed that a teacher needed to be positive, enthusiastic, patient, dedicated to the profession, have a love for kids, and be excited about discipline. Some teachers also commented that effective math teachers must also be organized, prepared, and have good classroom management. In addition to these comments, the respondents also suggested that effective teachers of mathematics should:

- Be able to engage students in math lessons;
- Make math lessons fun;
- Teach math concepts in multiple ways, and show multiple solutions to problems;
- Use discovery learning;
- Be willing to learn new instructional practices;
- Make students accountable for their own learning; and
- Be a learning and math role model.

When asked to consider the characteristics of talented math students, the respondents generated an exhaustive list of traits that they perceived that may be embodied by talented math students. The characteristics identified consistently by respondents included: self-confidence; motivation to learn; inquisitive; eager to communicate solutions; and a general excitement about mathematics. Other characteristics mentioned by the interviewees included: an ability to focus through chaos (a focus on the math instead of happenings in the classroom); persisting until challenging problems were solved; willingness to help one another and share solutions; sometimes derive solutions and cannot explain how; find and use patterns to solve problems; can follow multi-step problems without writing anything down; care about themselves and their work; and desire to be better and learn more about mathematics. Some teachers noted that talented math students might also be competitive, become displeased when they get the wrong answer, and at times are uncooperative with peers.

The topic of whether talented students were easy to identify and instruct produced conflicting responses from the interviewees. One teacher stated that talented math students "just stick out . . . you can just spot them." The remaining teachers however all agreed that talented math students were not easy to identify, and confidence levels, personality differences and gender were possible factors that made the identification of talent math students difficult. Two teachers asserted that using a variety of assessment procedures was important, as one teacher commented, "you need to use a variety of assessments continuously." When discussing how to instruct talented math students, most teachers agreed that catering to the needs of talented students is difficult. Most teachers suggested that talented math students required constant challenge, which can be difficult to achieve in the classroom. One teacher commented, "talented students can become bored and frustrated if challenge is not provided, and will lose interest." Another respondent, however, did not perceive instructing talented math students as problematic, and stated "I think they are easy to teach—they are willing to learn more, and are full of respect and determination." Most respondents indicated that they felt confident in teaching talented math students, however 2 teachers commented that their confidence was continuing to grow with experience and exposure to different resources. The following comment provides some insight into the complex challenge of instructing talented math students as perceived by one teacher:

> You have to be on top of your game. They will eat you alive; you need to know your stuff. You have to model that you don't have to know everything, so that they will accept that they don't have to be perfect . . . . This is especially important if gifted students are to develop the idea that there are multiple ways of solving problems, to share different approaches, and for enhancing the likelihood of creative solution. If you do not accept different answers, they will not respect you.

With the exception of a comment made by one teacher regarding the use of mentors and providing opportunities for extension, the instructional methods described by the teachers as being effective for fostering the academic needs of talented math students were the same as those used for general education students. Cooperative learning, using technology, utilizing project-based learning, and making connections between math content and the students' personal lives were among the responses forwarded by the teachers. Several teachers did mention the importance of providing challenge; however, they did not explain why this was important for talented math students or elaborate of specific techniques or examples of providing challenge.

Yale University

The 6 Yale University teachers were given surveys at the end of the after-school program to gauge their classroom style and determine what activities the students engaged in during the program. The activities teachers spent the most time on were reviewing assigned seatwork (10% - 30% of time), working problems with teacher's guidance (11% - 30% of time), and students working problems on their own without teacher's guidance (25% - 65% of time). Teacher's allowed at least 6-10 minutes for students to work on assigned coursework. All teachers sometimes or often had students work together in small groups and relate what they were learning in mathematics to their daily lives. All teachers thought that uninterested students sometimes or rarely limited how they taught their class. Low morale among students was also acknowledged as a limiting factor on how some of the teachers taught their class, but low morale did not influence the classroom more than sometimes.

The control classrooms rarely or never used calculators or computers for mathematical work, while the treatment or technology classrooms sometimes or often used calculators or computers for mathematical work. However, all teachers felt comfortable using technology with their students and felt it was important to use technology in their mathematics teaching. The teachers felt students were more motivated to learn mathematics when technology was involved. The teachers also believed that students think that mathematics is useful in everyday life.

Summary of Principals' Interviews

University of Connecticut

The University of Connecticut research team prepared a brief set of interview questions for the principals associated with the participating districts. The Principal's Guided Interview Form consists of 7 questions that ask for reflections on math instruction, characteristics of effective math teachers, descriptions of identifying and serving high potential math students, and methods for developing math talents.

When asked to consider the most important components of good math instruction, the School 1 principal focused the response on the unique needs of middle school students. This principal stated that "students of this age need to have links to what they are learning; connections to their life." The approaches must be sophisticated to ensure students' interest and responsiveness to a "balanced approach to fundamentals and group work," as well as independent activities.

The School 2 principal elaborated by supporting a

> constructivist approach where children are expected to make connections, experiment, and apply skills to challenging and relevant problems, and to provide explanations and not be afraid to fail.

Both principals emphasized the importance of making connections and learning links. Instructional activities that use examples from everyday life help students to see the value of mathematics and understand its usefulness. Good math instruction also uses assessment throughout lessons to gauge students' understanding.

Effective instruction with a dual emphasis on teaching and learning requires high quality teachers. Principals commented on the characteristics of an effective math teacher. Principals stressed the importance of working with teachers who have a clear picture of what is expected in the school. "Content knowledge is key" to effectiveness. Teachers need to be able to "break down steps in a process and share them with students, understand and apply knowledge of multiple intelligences, [and] able to integrate life into everyday situations."

School principal 1 said it is also critical for teachers in this district to have a "hunger to be adventurous." They need to have the "ability to reach students on different levels." Adventurous attitudes are supported by saying, "try it" to teachers and students. One example is an illustration of supporting teachers' creative approaches to teaching and learning. During an observation, a teacher outlined a map on the floor with tape. School principal 1 was thinking: "We just had the floors refinished!" These passing thoughts about the floor were eclipsed by the recognition that the "activity with the students was very engaging and, of course, I would support the teacher." The teacher took initiative to present the content in a different way and the students' responded positively.

Both principals described experiences with teaching high potential math students. School principal 2 noted that a high potential math student was "bored with traditional drill and practice, frustrated by being told you can only solve problems one way, abstract thinker." School principal 1 recognized the students' advanced skills and knew that the typical grade level curriculum would not be appropriate for the student's obvious skills and potential. As a teacher, School principal 1 met with the mathematics department head to discuss options that included developing curriculum for the student who comprehended and mastered material quickly. The principal provided a "menu of options" that included challenging materials, alternative chapters, and more demanding homework assignments to keep the student engaged. Principals described the characteristics of high potential math students such as those they worked with early in their careers as "natural intellect, curiosity, determination, and self-confident" (School principal 1); and independent, inquisitive, persistent, analytical" (School principal 2).

When asked about whether high potential math students are easy to identify and instruct, School principal 1 responded: Use "achievement data; put energy into kids who need to be identified." School principal 2 commented that it was not always easy to identify high potential math students: "Often they are non-compliant, don't conform to the steps/process." School principal 1 said it is important to "find students at high levels, and design ways to meet their academic needs."

When asked if high potential math students are easy to instruct. School principal 2 suggested it was important to "give them meaning and purpose, let them utilize or integrate all mediums, including technology, give them unlimited time." School principal 1 indicated that high potential students might not be easy to instruct; it "can go either way." Students who are not challenged, "may become bored-shut down" because they may have mastered the current content.

Meeting the academics needs of students of varying math abilities may not be easy. Middle school teachers and elementary school teachers may have the appropriate certification to work with the age group, but may not have the depth and breadth of the content area background to extend and enhance the student's math knowledge, skills, and abilities. School principal 1 indicated that the "background of teachers is important." Some K-6 teachers are "not comfortable with math."

To develop the talents of high potential math students (Question 7), School principal 2 offered several options: "Group with other talented math students and work in cooperative groups is one option, working alone with a mentor is another. Teach them skills when they need to know, otherwise let them explore solutions to problems." School principal 1offered several insights. Once again, teachers must "cater to age" to keep middle school students motivated. It is important to "challenge students to frustration level, stretch students' abilities." It is also critical to "celebrate success." It is important for this principal to encourage a learning environment in which peers recognize that "math academic achievement should be celebrated." Peer group needs to understand that "kids are smart" and that is "cool." The learning environment should promote an atmosphere in which students "feel admired by administrators and teachers." This attitude is supported by having "ongoing dialog with students." It is important to know what students are learning in school; wants to see midterm process; and "encourages students to try hard."

Administrators have many organizational, supervisory, and curricular responsibilities in schools. Throughout the implementation of the Algebra research study, the principals' support and awareness of the students' involvement with challenging curriculum was important to students and teachers. Students and teachers willingly responded to adding extra hours to their school week and maintained their engagement with the above grade level content and activities, and they understood the principals' willingness to ensure the program was successful.

Yale University did not administer the Principal's Guided Interview Form.

Classroom Observations

University of Connecticut

The University of Connecticut designed the Classroom Observation Scale (DeWet & Gubbins, 2006) for the Algebra pilot research study. The research team analyzed several existing instruments and formats to determine appropriate and efficient approaches to recording observational data. They also reviewed the *Teacher's Guide to Variables and Patterns: Introduction to Algebra,* which emphasizes the prerequisites of effective lessons. These prerequisites were incorporated into items for the Classroom Observation Scale to ensure a match between the rationale for teaching Algebra outlined in *Connected Mathematics 2* and the implementation of activities for this grade 7 unit:

- Provide clear and measurable objectives to help teachers and students understand the expectations for learning.
- Explain the purpose of the lesson to help students reflect on prior content and make connections with new learning.

- Emphasize students' understanding of the content and follow-up assignments.
- Promote connections to prior mathematical knowledge, skills, and understandings to orient students to old and new information.
- Think about and refine approaches to Algebra as the lessons build.
- Guide future lesson preparation and delivery by establishing what was learned and what needed to be learned.

Table 27 entitled Classroom Observation Scale provides a summary of 12 observations in Schools 1 and 2 from March to June 2006. The ratings of 1-4 represent each observer's interpretation of the effectiveness of the lesson. A quick overview of the observation results indicates the majority of observations resulted in ratings of moderately or very effective. These results will be presented by collapsing ratings of 3 and 4, moderately effective and very effective. Depending on the lessons and the timing of the observations, some items were not observable. Therefore, specific items were not rated. To accommodate this practice for data analysis purposes, the data in Table 27 include a column labeled "Not Observed."

The quality of the lessons was confirmed by the observers' selection of the highest rating of very effective on 9 of the 14 items. On items (1-3), ratings of moderately effective or very effective were selected at least 65% of the time across observations. The emphasis on presenting clear and measurable objectives was noted 67% of the time (Item 1). Teachers ensured students' understanding of lessons and assignments, as indicated by ratings of 75% (Item 2). Most of the time (75%) raters agreed that the teachers promoted connections to prior mathematical knowledge, skills, and understandings (Item 3). Ratings were somewhat lower for Item 4 (58%) related to the use of a variety of tools to reason together about Algebra.

In the next grouping of Items 5-8, ratings of moderately or very effective were chosen over 50% of the time. For Item 5, observers noted that teachers were moderately or very effective in engaging students in lessons 75% of the time. Fewer observations (59%) indicated high ratings for Item 6: Reflects on students' reactions to lessons. All but one observation confirmed that teachers were moderately or very effective in asking questions to press the students onward with solving the Algebra (91%) (Item 7). The ratings for Item 8 continued to be well above average with the selection of moderately or very effective in promoting communication about mathematics (75%).

Items 9-11, and 14 received ratings of moderately or very effective over 65% of the time. Teachers successfully engaged the intellect of students (83%) (Item 9), and they listened to students' comments and responses to assess their understanding of the content (66%) (Item 10). Observations revealed that teachers encouraged discourse about mathematical problems (67%) (Item 11). Teachers promoted a positive disposition toward mathematics, as indicated by 75% of the observations (Item 14).

Table 27

University of Connecticut Site: Classroom Observation Scale Ratings Across Schools

Items	1 Not Effective	2 Partially Effective	3 Moderately Effective	4 Very Effective	Not Observed
1 Provides clear and measurable objectives	8	17	50	17	8
2 Ensures that students understand lessons and assignments		8	33	42	17
3 Promotes connections to prior mathematical knowledge, skills, and understandings		17	17	58	8
4 Uses a variety of tools to reason together about Algebra	8	8	33	25	25
5 Engages students in lessons		8	17	58	17
6 Reflects on the students' reactions to lessons	8	8	17	42	25
7 Asks questions to press the students onward with solving the Algebra		8	8	83	
8 Promotes communication about mathematics		17	50	25	8
9 Engages students' intellect		8	25	58	8
10 Listens to students' comments and responses carefully to assess understanding	8	8	33	33	17
11 Encourages discourse about mathematical problems	8	8	25	42	17
12 Observes, listens to, and gathers information about students to assess their learning		25	25	25	25
13 Assesses students' mathematical knowledge and understanding formally	25	8	17	25	25
14 Encourages a positive disposition toward mathematics	8		8	67	17

Note: Table numbers are percentages.

Fewer observations (50%) resulted in the selection of moderately or very effective for Item 12: Observes, listens to, and gathers information about students to assess their learning. Item 13: Assesses students' mathematical knowledge and understanding formally received the lowest rating (42%) for moderately or very effective.

Table 27 documents ratings under each category of the response scale. Items rated as not effective (1) were selected 6 times, with one exception: Item 13: Assesses students' mathematical knowledge and understanding formally. This item may not have been an accurate assessment of the lessons, as implemented in the Algebra pilot research study. Even though the *Variables and Patterns* unit includes check up and partner quizzes, these were not used during the Algebra program. Item 12 also focuses on assessment: Observes, listens to, and gathers information about students to assess their learning. Ratings included: partially effective (25%), moderately effective (25%), very effective (25%), and not observable (25%).

The rating of partially effective (2) was chosen during the observations, indicating that certain sections of lessons needed more attention. Since the Classroom Observation Scale was used to document lessons, as opposed to collecting data to share with teachers on an ongoing basis, the ratings represent the observers' immediate interpretations of the lesson or part of the lesson being implemented during a specific timeframe. The opportunities and the timing of the observations affected the extent to which the content of all items could be observed in action. For 13 of the 14 items on the Classroom Observation Scale, 1-3 ratings were left blank.

Although there are limitations in capturing the dynamic interactions between and among teachers and students, and students and students, and then quantifying their interpretations in response to predetermined items, the observers completed field notes, which provided expanded perspectives on the Algebra research study.

Yale University did not implement the Classroom Observation Scale.

Summary of Classroom Observation Scale Ratings

The 14 close-ended items of the Classroom Observation Scale designed specifically for the Algebra research study provided quantitative perspectives of the effectiveness of the implementation of Investigations 1-4 from *Connected Mathematics 2: Variables and Patterns*. It is evident that observers agreed that lessons were moderately or very effective most of the time. Teachers and students maintained the philosophy of *Connected Mathematics 2* throughout the implementation of the unit.

Analysis of Field Notes

University of Connecticut

Field notes from the University of Connecticut research team provided descriptions of the Algebra program in process and documented the implementation of the curriculum unit, *Variables and Patterns*. Field notes offered insights into the Algebra program, as designed by the publishers of the *Connected Mathematics 2*, and the actual application of above grade level content in a setting other than the general education classroom. Field notes from Schools 1 and 2 are summarized below. As a result of multiple reviews of the field notes across 12 observations, two topics are presented to illustrate the Algebra research study in action. The teachers' use of questioning techniques and the emphasis on connections to tables, graphs, and equations are highlighted.

Students attended the Algebra program twice a week for 1½ hours, totaling 3 hours per week. Teachers welcomed students to their classes and engaged them with the above grade level curriculum. Small group, large group, and individual activities were used throughout lessons. The *Variables and Patterns* unit approached the development of students' understanding of Algebra concepts by presenting language-based stories with embedded numbers. Students would then depict the information in tables and graphs, as in the sample story problem.

Sample Story Problem

- *We started at 8:30 A.M. and rode into a strong wind until our midmorning break.*
- *About midmorning, the wind shifted to our backs.*
- *We stopped for lunch at a barbeque stand and rested for about an hour. By this time, we had traveled about halfway to Norfolk.*
- *Around 2:00 P.M., we stopped for a brief swim in the ocean.*
- *Around 2:30 P.M., we reached the north end of the Chesapeake Bay Bridge and Tunnel. We stopped for a few minutes to watch the ships passing. Because riding bikes on the bridge is not allowed, we put the bikes in the van and drove across.*
- *We took 7.5 hours to complete today's 80-mile trip.* (*Connected Mathematics 2: Variables and Patterns,* 2006, p. 14)

Eventually, the process would lead to the creation of formal, algebraic equations. This language emphasis allowed students to move from written details to a type of shorthand representing key descriptors within the story problem. As lessons progressed, the use of initial alphabetical letters as substitutes for words became more abstract with the introduction of x and y.

Sample Equations

Liz wonders whether they should rent a golf cart to carry the riders' backpacks at the park. The equation $c = 20 + 5h$ shows the cost c in dollars of renting a cart for h hours:

1. *Explain what information the numbers and variables in the equation represent.*

2. *Use the equation to make a table for the cost of renting a cart for 1, 2, 3, 4, 5, and 6 hours.*
3. *Make a graph of the data.*
4. *Describe how the pattern of change between the two variables shows up in the table, graph, and equation. (Connected Mathematics 2: Variables and Patterns,* 2006, p. 53*)*

Questioning Techniques

The CMP promotes questioning skills and the use of accurate mathematical terms. The *Teacher's Guide, Variables and Patterns,* provides a series of questions to guide students' thinking and encourage interactions between and among teachers and students.

Sample Questions

- *Which variable depends on the other?*
- *As the independent variable increases, what happens to the dependent variable? Does it increase or decrease?*
- *Will the increase or decrease be constant or will it slow down or speed up in some places?*
- *How will the change appear in the graph moving left to right?*
- *Is the graph likely to repeat in cycles?*

Ask students to defend their choices as you move from group to group. You are looking for reasonable interpretations, not for agreement on one graph as the "right" answer: This will tell you a lot about what sense students are making of how stories of change can be portrayed in graphs. (Connected Mathematics 2: Variables and Patterns, Teacher's Guide, 2006, p. 55*)*

The emphasis on language and its use and interpretation aided students in their realization that mathematics is a language of communication. Observers confirmed that teachers posed questions and asked for similar or different answers, as opposed to seeking one right answer only. "Students became animated as they explored different answers." The intent of this approach was to encourage students to defend their responses by explaining the mathematics and encouraging the use of appropriate terms. Observers commented on the probing questions. Teachers used the technique to "seek more and more information that leads thinking into possibilities." Teachers kept the "students on track." They did not take any random answer that was not even an educated guess. They focused the possible answers by offering clarifying questions. Teachers posited different scenarios related to the problem as a way to hone in on the solution. Students enjoyed these "what if kinds of scenarios." Such scenarios "push{ed} students to think deeper and further." Teachers "supported progressive thinking-So, if this is true, what then?" They encouraged responses and used phrases such as "Good thinking!" "Great idea!"

Teachers also asked guiding questions to determine students' understanding of the content. *(This is an instance where the quantitative data do not mirror the field notes.)* Teachers wanted to be sure that students were processing the content appropriately. Throughout the lessons, teachers asked defining questions: "Why do we use labels?" "What is the independent variable?" "So, which of these data would be the independent variable?" Establishing and reinforcing the use of terminology helped students to discuss the content "like young mathematicians."

One observation documented how students also adopted the process of asking questions. The observer stated that one student "kept asking questions to check her reasoning, and kept at it until she finally exclaimed, 'Oh, now I get it!' Then she quickly finished the work."

To encourage student discourse, the teachers connected students' questions to those posed by others to promote, probe, and support ideas. Students' responses were met with positive reinforcement and, at times, humor to encourage them to rethink or clarify responses. The repartee among teachers and students provided a safe environment to take a risk with an answer that may not be accurate at first. To ensure that students were not reluctant to offer somewhat incomplete answers that would help teachers gauge their students' level of understanding, one teacher used the following tactic. The teacher asked, "Give me a percentage of how sure you are of your answer?" This was a good way to promote the student's reflection on the initial answer. The teacher would encourage a variety of answers without responding yes/no until one student arrived at a definitive answer. For example, one teacher asked questions about making a table.

> "In making a table, do we do this horizontally or vertically?" Students chose horizontally.
>
> "What variable do we put on top?" Students give various answers.
>
> One student says: "Does it matter?"
>
> Teacher says enthusiastically: "Yes, yes. That's the question. Does it matter?"
>
> Students respond with various answers. Teacher says: "How sure are you?"
>
> "Can we get the same answer with either variable on top?"

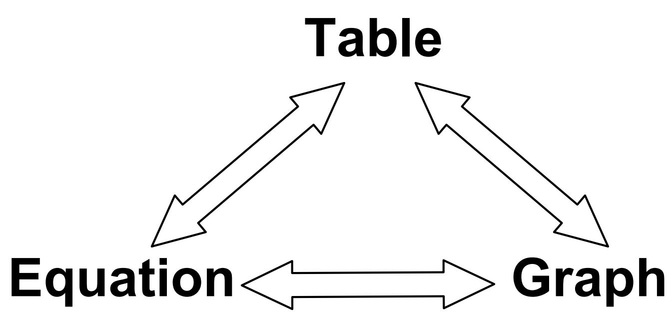
Students had great fun coming up with answers to questions. They were eager to answer first and they said "Yeah" when they were called upon to offer their answer. When students gave a good answer, one teacher says: "Great. You are right! I guess you learned something."

When teachers posed questions, most students were eager to answer, as indicated by raising their hands. When they did falter with their answer, teachers provided immediate feedback. If the answer was close, but not entirely correct, one teacher would repeat the answer to indicate the student needed to rethink the answer. One student said to another one who kept offering the same answer. "She's telling you that you don't have the answer."

Students learned the rules of discourse. Teachers and students were in a supportive atmosphere that allowed them to offer their ideas readily. They all respected each other's willingness to engage in solving problems and "staying with unfinished learning" until solutions were reached in this positive mathematical learning environment.

Connections to Tables, Graphs, and Equations

Teachers asked students to reflect on what they had learned in a prior lesson focusing on writing equations to represent situations. Students noted how they used tables, graphs, and equations. Some equations included words (distance = rate x time) and others used letters to represent the variables (d/t = r). Students understood the connections between and among tables, graphs, and equations. One teacher drew a diagram to illustrate the connections.



Students reminded the teacher that there were language-based equations such as the one above—distance = rate x time. Students were asked if they knew how many ways you could express the d = rt equation correctly. After several answers and questions, they were shown that there are three equations related to this specific concept:

1. d/t = r
2. d = rt
3. r = t/d

Teachers held high standards for students' work and how they represented data in graphs, tables, or words. Some students were quite surprised that one teacher actually wrote some non-technical terms (e.g., stuff, things) on the board as students brainstormed what they learned. One student was so amazed that the teacher wrote down the words students offered, he said: "I use sophisticated words." His humor was greeted positively by all.

Investigations 1-3 in *Variables and Patterns* were completed without technology in 2 of the 5 classes involved in this Algebra research study. The non-technology classes used paper, pencil, and drawings of tables and graphs on the chalkboard. The technology classes used Excel spreadsheets and graphing calculators starting with Investigation 1. All students had access to the graphing calculators when they reached Investigation 4.

During one observation, students were applying their knowledge in Investigation 1 to their ability to interpret a record of a child's growth from birth to age 18. The table in their book included two columns: Age (yrs) and Height (in.). At birth, height was recorded as 20 inches. Students were involved in the following tasks and questions:

a) Make a coordinate graph of Katrina's height data.
b) During which time interval(s) did Katrina have her greatest "growth spurt?"
c) During which time interval(s) did Katrina's height change the least?
d) Would it make more sense to connect the points on the graph? Why or why not?
e) Is it easier to use the table or the graph to answer parts (b) and (c)? Explain. (*Connected Mathematics 2, Variables and Patterns,* 2006, p. 17)

When students were asked to respond to question (c) about the time interval in which Katrina's height changed the least, they chose 0.5 increases over no increase (ages 14-16). One student, in particular, was convinced that no change was incorrect. He thought the answer could not qualify for "changed the least." He explained his comments and other students helped him reason through his initial response. To him, change meant the height could not be stable over time. With further probing questions, the student was able to reorient his thinking.

Students used their graphing calculators and made a couple of plots, and they interpreted the data related to the questions a-e. The observer commented that students were "getting the point about data in a table vs. data in a graph and when one is actually more informative than the other."

Students were still trying to master the use of graphing calculators in Investigation 1. One observer noted:

> When they were asked to create a list for their calculator, several did not know that the first column of numbers in the table is the x-axis. They don't know x—independent vs. y—dependent variable. They seem to understand "coordinate pairs."

Throughout Investigations 1-4, students became more flexible in their use of tables and graphs. Each investigation is followed by "ACE," which are Applications, Connections, and Extensions of what they are learning. In one observation of a teacher's class that used technology throughout the program, the observer recorded the following field notes for an Application problem related to a bike tour box lunch costs. Students were asked to write an equation based on a data table and find the lunch cost for 25 riders. They were also asked to determine how many riders could eat lunch for $89.25.

- Students were assigned to complete 4b and 4c individually. All students able to find answer.
- Reviewed both answers as a class.
- Some students went right to the calculator to find answer and others did multiplication by hand for 4b.
- Most students divided to find answers for 4c but at least 1 used the guess-and-test strategy.

Students and teacher continued with a Connection problem: Find the indicated value or values: "the 10th odd number (1 is the first odd number, 3 is the second odd number, and so on)" (*Variables and Patterns,* 2006, p. 59). Observer notes:

- Some students got answer quickly; others confused by concept of 10th odd number.
- Some created a table, others listed odd numbers on calculator did 1 + 2, then kept +2 until 10th.
- [Teacher] created table on board.
- Asked for 100th odd number.
- Looked for pattern in numbers:
  -difference b/t 2 variables grows by 1 each time
  -n + (n-1) = nth # - use pattern to find that the 100th is 199
  -n x 2 – 1 = nth #.
- Can you write an equation for that? Did together as a class.
- What is 500th? 999
- Would it be a good idea to make table? No—use equation.

Yale University did not provide summaries of observations.

Summary of Field Notes

Field notes captured segments of Investigations 1-4, discussions, and application of knowledge and skills. Students and teachers were receptive to their involvement in the Algebra research study. The grade 6 students spent an additional 3 hours per week to explore words, tables, and graphs and how these approaches led to algebraic understanding.

Conclusions

University of Connecticut

The University of Connecticut schools for the research study entitled Unclogging the Mathematics Pipeline Through Access to Algebraic Understanding involved grade 6 students for 30 hours of after-school instruction in Algebra. Given the sample size (73 students), data could not be disaggregated by Intervention 1 (technology) and Intervention 2 (non-technology). Therefore, the following questions represent a modification from those stated in the original research proposal.

1. Does involvement in an after-school Algebra pilot research study impact students' mathematics achievement?
2. Does participation in the mathematics intervention affect student self-efficacy, and positive attitudes and interest in mathematics?
3. What are students' perceptions of the mathematics classroom practices in the after-school pilot research study?
4. What are teachers and administrators' perceptions of teaching and learning mathematics?

Grade 6 students who earned at least a B in mathematics were eligible for participation in the screening process for the pilot research study, which included the ITBS, Mathematics Problem Solving and Data Interpretation (grade 8) subtest and the IAAT (grade 8). Of the 110 students, 73 participated in the program. The after-school pilot research study occurred for 10 weeks (1½ hours, twice a week) with 30 students working with 2 teachers in School 1, and 43 students with 3 teachers in School 2. Teachers used *Connected Mathematics 2, Variables and Patterns,* a unit typically designed for grade 7 students.

To investigate the impact of the after-school pilot research study emphasizing algebraic understanding, Research Question 1 addressed student achievement. Achievement data were analyzed across schools and by schools. There were statistically significant differences between the pre and posttest scores across schools and by schools on the ITBS, Mathematical Problem Solving and Data Interpretation subtest; IAAT; and the CMP unit tests (both versions-12 items and 15 items). Students involved in the Algebra research study improved their mathematics achievement during the after-school pilot research study. The mastery of the content related to algebraic understanding was evident as students' became more adept in creating and interpreting tables, graphs, and equations.

Participation in the Algebra research study did not affect students' self-efficacy, and positive attitude and interest in mathematics. Statistical differences were not evident in analyzing the pre/posttest results of Research Question 2. Students were positive about mathematics before and after their involvement in the after-school pilot research study. They added an additional 30 hours beyond their school day to study math, which probably corroborates their self-efficacy and positive attitudes and interest in mathematics. Of the students in School 1, 87% had perfect attendance, and in School 2, 65% had perfect attendance. Their self-efficacy, positive attitude, and interest in mathematics were also supported by their comments on the Student Questionnaire.

The results for Research Question 3, focusing on students' perceptions of the mathematics classroom practices in the after-school program, indicated that the majority found the intensive Algebra program fun, interesting, and exciting. Many noted that the work differed from the regular classroom because it was more difficult. Yet, the students in this study found hard, difficult, and challenging work in Algebra to be fun and exciting.

In response to Research Question 4, teachers and administrators shared their perceptions of teaching and learning mathematics through a series of interview questions. They recognized the importance of effective instruction in mathematics and were familiar with the characteristics of mathematically talented students. Challenging these students was important to the continuation of their learning.

Classroom observations provided a complete perspective on the pilot research study as planned and as implemented. These observations confirmed teachers and students' adherence to the philosophy of the CMP, and documented students' ability to understand and apply advanced-level knowledge and skills related to algebraic understanding. Grade 6 students spent an additional 3 hours per week to explore words, tables, and graphs and how these approaches led to algebraic understanding. The selected curriculum, *Connected Mathematics 2, Variables and Patterns,* provided a format and guide. However, the dynamics within the classes were definitely determined by the teachers and students' commitment to learning how to think algebraically. Students mastered above grade level content and concepts and achieved representational fluency, which is "students' ability to solve problems using tables, graphs, words, or symbolic representations" (Wisconsin Center for Education Research, 2006, p. 3). Algebraic reasoning prepares students for future accomplishments in mathematics, and the 73 students and their 5 teachers at the University of Connecticut research sites were certainly successful in achieving the goals of this pilot research study.

Implications for Classroom Practices

Gauging students' potential in mathematics is not a common practice in middle schools. Accessing students' knowledge and skills with tests that were 2 years above grade level provided perspectives on what students know and do not know. Students understood that they might encounter problems in the ITBS, Mathematical Problem Solving and Data Interpretation subtest and the IAAT that were unfamiliar. They were encouraged to reason through the problems to provide broader views of skills and abilities that would not be available through grade level assessments. The 73 students who participated in the Algebra research study were eager to learn Algebra. The results of the after-school intervention illustrated the impact of 30 hours of involvement with content that was slightly familiar to some and unfamiliar to many. If we truly want to unclog the mathematics pipeline and encourage students to pursue algebraic reasoning, then we must finds ways to integrate the necessary content and skills into the daily curriculum.

Yale University

To compare the pre/posttest differences across schools and across interventions, paired samples *t* tests were conducted on each matched set of achievement data: ITBS pre/post; IAAT pre/post; and CMP pre/post (12 items); CMP pre/post (15 items). For the overall population examined, ITBS scores dropped slightly from pre to posttest, while the IAAT and CMP (12 and 15 items) scores all increased.

In addition to examining the overall achievement scores, analyses of the results across treatment type were performed. The mean difference indicated increases in IAAT and CMP (12 and 15), while the ITBS scores remained approximately the same, in both the technology (treatment) and no additional technology (control) cases. The gains were not significantly different in either treatment case.

Further analyses resulted in paired samples *t* tests being performed where treatment type and gender were both considered. The changes in the mean IAAT, CMP 12, and CMP 15 pre and post scores were different for female and male participants in the two conditions. The presence of technology (treatment) increased the mean female scores more than the mean male scores, while the no additional technology condition (control) resulted in a greater rise in mean male scores than mean female scores. The results for the ITBS assessment were different than the other three assessments. The mean ITBS scores of females in the technology condition and males in the control condition actually decreased.

Students completed the Attitudes Toward Mathematics (Tapia) survey on a pre and posttest basis to help determine if involvement with after-school pilot research study affected students' self-efficacy and positive interest and attitudes in mathematics. The score for the instrument was the sum of all ratings. The mean pretest score for all students participating in the Yale Algebra research study was 139.89, and the mean posttest score was 141.33.

The data were also analyzed by condition and gender. There were no significant changes between pre and posttest attitudes toward mathematics in either condition or gender.

The results that proved interesting occurred when analysis was performed across gender and condition together. Females in the control group showed a small decrease in math attitude, while male's math attitude increased by 6 points in the control group. Meanwhile, the mean math attitude score of females in the treatment group increased by nearly 3 points and males in the treatment group decreased by nearly 3 points.

Participation in the Algebra research study did not affect students' self-efficacy, and positive attitude and interest in mathematics. Students were positive about mathematics before and after their involvement in the after-school pilot research study. Their perceptions of the mathematics classroom practices in the after-school program indicated that there were mixed opinions, students liking some aspects of the program and not others. There was no consistent agreement on the difference between regular classroom practices and the after-school program.

Unclogging the Mathematics Pipeline Through Access to Algebraic Understanding

Section B: University of Virginia Algebra Pilot Research Study

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ABSTRACT

This pilot research study attempted to determine whether varying the form in which mathematical material is presented creates greater equality of opportunity. The particular mathematical material studied comprised types of Algebra word problems that typically are presented in the logical/mathematical mode and that utilize spatial visualization (e.g., mixture, work, and time-rate-distance problems). The design investigated whether presenting such material in a narrative mode with spatial aids can equalize opportunities for mathematical achievement in Algebra. The ultimate goal of the project was to increase students' math achievement and students' attitudes toward and interest in mathematics.

Section B: University of Virginia Algebra Pilot Research Study summarizes data from the summer intervention program. Some of the quantitative data presented include students' scores from a subset of the Yale University data. All of the qualitative results are from the University of Virginia dataset only.

The following research questions guided the pilot research study:

- Do students who participate in the mathematics intervention outperform control students on a measure of mathematics achievement after taking into account pretreatment mathematics achievement differences?
- Do students who participate in the mathematics intervention outperform control students on a measure of mathematics achievement after taking into account pretreatment Algebra aptitude differences?
- What are students' perceptions of the mathematics classroom practices in the mathematics intervention?

Although there were no statistically significant differences between treatment and control groups on achievement, aptitude, or attitudes, three important findings emerged from the qualitative data that merit consideration. Each will be considered separately below.

1. One interesting finding emerging from the study was that all teachers unanimously expressed liking being provided with a prescribed curriculum that was easy for them to follow. All perceived the curriculum to be high-level, challenging, and engaging for students, as well as enjoyable to teach. As a result, all teachers maintained a high level of fidelity to the treatments.
2. Students in both the treatment and control groups expressed thoroughly enjoying the math program. Students from both groups cited the small class size, the "fun" and interactive math activities, and the high level of challenge as the primary reasons for enjoying the program. None mentioned the technological components as contributing to their enjoyment of or engagement in the program. This is interesting in light of recent attention focused in the literature on the use of technology to engage students in learning math.
3. Students in the study indicated a clear preference for learning at a faster pace and at greater levels of challenge than they normally got the opportunity to do in their regular math classes. Nearly all of the participating students indicated that they learned *better* under the conditions of a quickened pace and increased challenge.

The findings suggest that while technology provides a useful pathway to understanding for students, it alone does not necessarily encourage or ensure student engagement. Instead, it seems that for the students in this study at least, high-level challenge, one-on-one time with the teacher, and hands-on activities are what is needed to engage advanced students in learning math.

Nearly all participating students indicated an eagerness to learn more math than they were able to do during their regular school year classes. This signals a need for a consideration of the match between the challenge level of the mathematics curriculum offered in our middle classrooms and the needs and abilities of the adolescents populating these classrooms. It begs the questions, *Are we underestimating the level of mathematical ability and interest of many of our middle school students? Are we limiting the possibilities for able math students by the lack of fit between the curriculum and instruction offered in our middle school math classes and their mathematical abilities and interests?*

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EXECUTIVE SUMMARY

This pilot research study attempted to determine whether varying the form in which mathematical material is presented creates greater equality of opportunity. The particular mathematical material studied comprised types of Algebra word problems that typically are presented in the logical/mathematical mode and that utilize spatial visualization (e.g., mixture, work, and time-rate-distance problems). The design investigated whether presenting such material in a narrative mode with spatial aids can equalize opportunities for mathematical achievement in Algebra. The ultimate goal of the project was to increase students' math achievement and students' attitudes toward and interest in mathematics.

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- Do students who participate in the mathematics intervention outperform control students on a measure of mathematics achievement after taking into account pretreatment mathematics achievement differences?
- Do students who participate in the mathematics intervention outperform control students on a measure of mathematics achievement after taking into account pretreatment Algebra aptitude differences?
- Do students who participate in the mathematics intervention have higher attitudes toward mathematics than control students after taking into account pretreatment attitude differences?
- What are students' perceptions of the mathematics classroom practices in the mathematics intervention?

Site 1. Two schools were randomly assigned to the control and treatment conditions for Site 1. For each school, there was a treatment teacher and a control teacher. The school district participating requested that the study be conducted during the month of July as that was their regular summer school program. Four teachers (2 from each school) were trained during a 2-day training workshop with individuals knowledgeable about *Connected Mathematics* curriculum from the University of Connecticut for a total of 12 hours of training.

Site 2. All 6 teachers associated with the Yale University pilot research study participated in a 1-day workshop on the *Connected Mathematics Program* (CMP), lead by an outside consultant who had provided professional development on the CMP for the State of Connecticut for many years. The after-school pilot research study occurred for 10 weeks (1½ hours, twice a week).

Quantitative data from 83 students were analyzed (41 control group; 42 experimental group). Of these students, some students were from the Site 1: University of Virginia summer pilot research study. A subset of data from Yale University's after-school pilot research study (Site 2) was included in the quantitative analyses for Section B of this research monograph.

All sites used the following measures: (a) the eighth grade level of the Problem Solving and Data Analysis subtest of the Iowa Tests of Basic Skills (ITBS) achievement test; (b) the Iowa Algebra Aptitude test; (c) the Tapia Attitudes toward Mathematics; and (d) the unit test associated with the CMP unit being taught. The ITBS, the Iowa Aptitude assessment, and the attitudes toward mathematics assessments were given during the school year as part of the identification of potential students. The end of unit assessment was given twice during the course of the project; prior to instruction and at the completion of the unit.

Units from the *Connected Mathematics* series related to linear equations were utilized as the basis for the treatment curriculum. The treatment curriculum taught the same skills as those taught in the *Connected Mathematics* unit on *Variables and Patterns*, but supplemented the lessons with technology-assisted instruction designed to facilitate student understanding of presented concepts.

The *Variables and Pattern* unit was taught during 12, 4-hour sessions over the course of 3 weeks in two classrooms during summer school (University of Virginia) or 3 hours per week for 10 weeks during the school year (Yale University). Classes were taught by teachers trained in the use of the curriculum. Instruction in the treatment classes was delivered via narrative and technology-assisted instruction, while instruction in the control classes was delivered via narrative and spatial instruction only.

Quantitative Results

A between-subjects analysis of covariance was performed on mathematics achievement. The independent variable was treatment status (treatment or control) for each research question. Covariates were pre-measures for the ITBS (research question 1), the Algebra aptitude (research question 2), and attitudes (research question 3). The dependent variable for questions 1 and 2 was the end-of-unit post assessment and for question 3 was the post attitudinal measure.

Research Question 1: Do students who participate in the mathematics intervention outperform control students on a measure of mathematics achievement after taking into account pretreatment mathematics achievement differences?

After adjustment by the ITBS covariate, there was no statistical difference between the treatment group and the control group on the end-of-unit assessment (power=.49).

Research Question 2: Do students who participate in the mathematics intervention outperform control students on a measure of mathematics achievement after taking into account pretreatment Algebra aptitude differences?

After adjustment by the Algebra aptitude covariate, there was no statistical difference between the treatment group and the control group on the end-of-unit assessment (power=.51).

Research Question 3: Do students who participate in the mathematics intervention have higher attitudes toward mathematics than control students after taking into account pretreatment attitude differences?

After adjustment by the pretreatment attitude covariate, there were no statistical differences between the treatment group and the control group on any of the four attitudinal subscales.

Qualitative Results

Qualitative data are from Site 1, which is the University of Virginia summer Algebra intervention program only.

Three important findings emerged from the qualitative data that merit consideration. Each will be considered separately below.

1. One interesting finding emerging from the study was that all teachers unanimously expressed liking being provided with a prescribed curriculum that was easy for them to follow. All perceived the curriculum to be high-level, challenging, and engaging for students, as well as enjoyable to teach. As a result, all teachers maintained a high level of fidelity to the treatments. This suggests that, to encourage treatment fidelity in studies asking teachers to implement a certain type of curriculum, a highly prescribed, scripted curriculum may be most effective.

2. Students in both the treatment and control groups expressed thoroughly enjoying the math program. Students from both groups cited the small class size, the "fun" and interactive math activities, and the high level of challenge as the primary reasons for enjoying the program. None mentioned the technological components as contributing to their enjoyment of or engagement in the program. This is interesting in light of recent attention focused in the literature on the use of technology to engage students in learning math. However, the students in this study—whether provided with technology as a learning tool or not—indicated that raising the challenge level of the content, providing hands-on activities, and providing more intimate learning environments with opportunities for one-on-one discussions with the teacher may be the keys to increased student engagement in and enjoyment of math.

   The treatment group students did indicate that the graphing calculators were useful tools in helping them to learn math. Control students also indicated that the calculators, when they were distributed to them at the end of the program, were helpful in learning math.

   Taken together, these two findings suggest that while technology provides a useful pathway to understanding for students, it alone does not necessarily encourage or ensure student engagement. Instead, it seems that for the students in this study at least, high-level challenge, one-on-one time with the teacher, and hands-on activities are what is needed to engage advanced students in learning math.

3. Students in the study indicated a clear preference for learning at a faster pace and at greater levels of challenge than they normally got the opportunity to do in their regular math classes. Nearly all of the participating students indicated that they learned *better* under the conditions of a quickened pace and increased challenge. Again, nearly all participating students indicated an eagerness to learn more math than they were able to do during their regular school year classes. This signals a need for a consideration of the match between the challenge level of the mathematics curriculum offered in our middle classrooms and the needs and abilities of the adolescents populating these classrooms. It begs the questions, *Are we underestimating the level of mathematical ability and interest of many of our middle school students? Are we limiting the possibilities for able math students by the lack of fit between the curriculum and instruction offered in our middle school math classes and their mathematical abilities and interests?*

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Methodology

The following research questions guided the pilot research study:

- Do students who participate in the mathematics intervention outperform control students on a measure of mathematics achievement after taking into account pretreatment mathematics achievement differences?
- Do students who participate in the mathematics intervention outperform control students on a measure of mathematics achievement after taking into account pretreatment Algebra aptitude differences?
- Do students who participate in the mathematics intervention have higher attitudes toward mathematics than control students after taking into account pretreatment attitude differences?
- What are students' perceptions of the mathematics classroom practices in the mathematics intervention?

Participants

Site 1. Two schools were randomly assigned to the control and treatment conditions for Site 1. For each school, there was a treatment teacher and a control teacher. Care was taken to ensure that there was no treatment contamination through the observations of classrooms.

Site 2. Originally 3 school districts were contacted for participation. Two superintendents were agreeable to work with us, and put us in touch with their math specialists, who then approached teachers in their district. We worked with 4 teachers in 2 schools in District 1; 2 teachers in 1 school in District 2. Within each school, there was 1 technology and 1 non-technology teacher.

Context of Sites

Site 1. The school district participating requested that the study be conducted during the month of July as that was their regular summer school program. They also did not want to interfere with instruction occurring during the school year as it had implications for the state assessments. Therefore, during the month of May, all students in the standard level curriculum (i.e., not special education and not gifted) were assessed and all students were invited to participate per the district request.

Four teachers (2 from each school) were trained during a 2-day training workshop with individuals knowledgeable about CMP curriculum from the University of Connecticut for a total of 12 hours of training. At the time of training, teachers were not yet assigned as "treatment" or "control" teachers; as a result, the same training was received by all 4 teachers. Training focused on familiarizing the teachers with the program curriculum, technology, and supplemental resources. Upon completion of training, teachers were randomly assigned to treatment groups with one teacher in each school an experimental teacher and one a control teacher.

Site 2. All 6 teachers associated with the Yale University pilot research study participated in a 1-day workshop on the CMP, lead by an outside consultant who had provided professional development on the CMP for the State of Connecticut for many years. The after-school pilot research study occurred for 10 weeks (1½ hours, twice a week).

Measures

All sites used the following measures: (a) the eighth grade level of the Problem Solving and Data Analysis subtest of the ITBS achievement test; (b) the Iowa Algebra Aptitude test; (c) the Tapia Attitudes toward Mathematics; and (d) the unit test associated with the CMP unit being taught. The ITBS, the Iowa Aptitude assessment, and the attitudes toward mathematics assessments were given during the school year as part of the identification of potential students. The end of unit assessment was given twice during the course of the project; prior to instruction and at the completion of the unit. Tables 28-30 provide descriptive statistics.

Table 28

Descriptive Statistics for Pretreatment Achievement/Aptitude Measures

Condition	ITBS			Aptitude		
	$\bar{X}$	sd	N	$\bar{X}$	sd	N
Control	66.56	18.86	41	64.12	16.48	41
Experimental	68.07	20.80	42	64.57	22.53	42

Table 29

Descriptive Statistics for Pre-attitude Assessment

	Self-confidence			Value			Enjoyment			Motivation		
	$\bar{X}$	sd	N	$\bar{X}$	sd	N	$\bar{X}$	sd	N	$\bar{X}$	sd	N
Control	63.05	12.92	22	42.53	9.31	19	39.00	8.76	22	20.14	4.74	21
Experimental	65.03	6.78	35	43.59	4.25	37	39.88	6.13	34	20.14	3.02	37

Table 30

Descriptive Statistics for Pre-unit Assessment: Site 2

Condition	Unit Test		
	$\bar{X}$	sd	N
Control	7.45	2.92	33
Experimental	6.74	2.78	34

Note: Site 1 did not administer the pre-unit test

Curriculum Development

Units from the CMP series related to linear equations and not modified in any way were chosen as the control curriculum. A team of math experts utilized these units as the basis for the treatment curriculum, adjusting the lessons to teach the same skills as the control curriculum, but supplementing it with technology-assisted instruction (e.g., use of graphing calculators and data projectors during instruction). Once the treatment curriculum was developed, sample lessons from both the treatment and control conditions were submitted to a panel of secondary mathematics educators (5 in Connecticut and 5 in

Virginia) asking for their evaluation and feedback on the quality of the lessons and their ability to classify the lessons according to the two interventions.

Upon receipt of the feedback, modifications of the lessons were made and then piloted with 40 students in grades 7 and 8 (20 in Connecticut and 20 in Virginia). Students were observed as they worked on the sample lessons and written feedback was requested from pilot teachers and students about the quality of the lessons and the extent to which the different presentations of algebraic thinking helped students process the content.

Treatment Classes

Units from the *Connected Mathematics* series related to linear equations were utilized as the basis for the treatment curriculum. The treatment curriculum taught the same skills as those taught in the *Connected Mathematics* unit, but supplemented the lessons with technology-assisted instruction designed to facilitate student understanding of presented concepts. Final lessons focused on linear equations involving mixture, motion and work problems were taught during 12, 4-hour sessions over the course of 3 weeks in 2 classrooms during summer school (University of Virginia) or 3 hours per week for 10 weeks during the school year (Yale University). Classes were taught by teachers trained in the use of the curriculum. Instruction in the treatment classes was delivered via narrative and technology-assisted instruction.

Control Classes

Units from the *Connected Mathematics* series related to linear equations comprised the curriculum for the control treatment. These lessons on linear equations involving mixture, motion and work problems were taught during 12, 4-hour sessions over the course of 3 weeks in 2 classrooms during summer school (University of Virginia) or 3 hours per week for 10 weeks during the school year (Yale University). Instruction in the control classes was delivered via narrative and spatial instruction only.

Data Collection

Quantitative Data

Pretreatment. Students were administered three measures: (a) an out-of-level achievement test (ITBS grade level 8); (b) the IAAT; and (c) the Tapia, an instrument designed to measure students' attitudes toward mathematics.

Posttreatment. Students were re-administered the three instruments that were given prior to participating in the study and in addition, students were administered the end of the unit assessment that corresponded with the unit that was taught as part of the study.

Site 1: Qualitative Data

To address students' perceptions of the mathematics classroom practices, data were collected through a survey at the conclusion of the course that included open-ended questions probing students' perceptions of their experiences with the curriculum.

To ensure fidelity to the treatment, data were collected through the following methods:

Observations. Each class (both control and treatment) was observed at least three times a week for an hour (for a total of at least 9 hours per class) by a trained observer using the Unclogging the Mathematics Pipeline Classroom Observation Scale (De Wet & Gubbins, 2006) to determine the extent to which the teacher utilized the curriculum appropriately and to describe teacher behaviors (such as "provides clear and measurable objectives; uses a variety of tools to reason together about Algebra; reflects on students' reactions to lessons"). The observation scale was developed from the National Council of Teachers of Mathematics Professional Standards for Teaching Mathematics. The observation scale utilized a 5-point Likert Scale (*strongly disagree, disagree, neutral, agree, strongly agree*) and also allowed room for general observations and comments.

Teacher Logs. Teachers were asked to maintain logs to record reactions to lessons and provide evidence of how the implementation matched the specific intervention.

Data Analysis

Quantitative Data

A between-subjects analysis of covariance was performed on mathematics achievement. The independent variable was treatment status (treatment or control) for each research question. Covariates were pre-measures for the ITBS (research question 1), the Algebra aptitude (research question 2), and attitudes (research question 3). The dependent variable for questions 1 and 2 was the end-of-unit post assessment and for research question 3 was the post attitudinal measure. Analyses were performed with SPSS 15.0 Windows, weighting cells by their sample sizes and adjusting for unequal n.

Site 1: Qualitative Data

To make sense of the large amount of data resulting from the students' responses to open-ended survey questions, qualitative data were analyzed using an inductive approach to analysis (Patton, 1990). In inductive analysis, patterns, themes, and categories of analysis emerge from the data rather than being imposed prior to data collection and analysis (Patton, 1990, p. 390). As patterns and themes emerged during data coding, categories were developed. Categories were then refined, collapsing seemingly overlapping categories and renaming as necessary. From these categories, the study results were derived.

Results

Quantitative Data

Quantitative data from 83 students were analyzed (41 control group; 42 experimental group). Of these students, some of the students were from the Site 1: University of Virginia summer pilot research study. A subset of data from Yale University's after-school pilot research study (Site 2) was included in the quantitative analyses for Section B of this research monograph.

First, assumptions of normality of sampling distributions, linearity, homogeneity of variance, homogeneity of regression, and reliability of covariates were evaluated. Results indicated all assumptions were met satisfactorily. Tables 31 presents the descriptive statistics for the post-unit assessment.

Table 31

Descriptive Statistics for Post-unit Assessment

Condition	Mean	Std. Dev.	N
Control	9.34	2.65	41
Experimental	8.45	2.66	42
Total	8.89	2.68	83

Research Question 1: Do students who participate in the mathematics intervention outperform control students on a measure of mathematics achievement after taking into account pretreatment mathematics achievement differences?

After adjustment by the ITBS covariate, there was no statistical difference between the treatment group and the control group on the end-of-unit assessment (power=.49).

Research Question 2: Do students who participate in the mathematics intervention outperform control students on a measure of mathematics achievement after taking into account pretreatment Algebra aptitude differences?

After adjustment by the Algebra aptitude covariate, there was no statistical difference between the treatment group and the control group on the end-of-unit assessment (power=.51).

Research Question 3: Do students who participate in the mathematics intervention have higher attitudes toward mathematics than control students after taking into account pretreatment attitude differences?

After adjustment by the pretreatment attitude covariate, there were no statistical differences between the treatment group and the control group on any of the four attitudinal subscales.

Site 1: Qualitative Data

Qualitative data are from Site 1, which is the University of Virginia summer Algebra intervention program only.

Fidelity to the Treatment

Classroom observations and teacher logs indicated that the 2 treatment teachers were largely successful in implementing the treatment with fidelity; neither teacher was observed making any significant adjustments to or modifications of the curriculum. In fact, both teachers expressed liking having a very directed and detailed set of lesson plans from which to work, as they believed it saved them a great deal of planning time. One teacher commented to an observer, "I don't look at this at all over the weekend. I can just come in on Monday and teach right from the script." Because these sites were implementing the treatment during summer school, both treatment teachers expressed pleasure at not having to do any planning overnight. Both teachers expressed a desire to have curriculum like this all year.

While both treatment teachers were successful in following the steps in the lessons, this did not ensure that they rated high on the *Classroom Observation Scale*. One teacher followed the curriculum closely, but did not ask many questions of students beyond those elucidated in the lessons and did not push students to make connections to prior learning. The second treatment teacher expressed on several occasions to an observer that she felt there were many students taking the class who were not "up to this level," but made no attempts to accommodate those learners. She provided help to those students when they asked for it, but did not otherwise make adjustments for them.

Both treatment teachers, however, were impressed by how engaged the students seemed in the curriculum and the activities. Both teachers noted in their logs that the students enjoyed using their calculators and had mastered them quickly. Both teachers believed that the graphing calculators, in particular, provided students with important entry points to understanding math.

The control teachers were as vocal as the treatment teachers about how much they like the *Connected Mathematics* series and, in particular, the ease with which a teacher could follow and implement the lessons. Like the treatment teachers, the control teachers stuck closely to the outlined lessons.

Research Question: What are students' perceptions of the mathematics classroom practices in the mathematics intervention?

Overwhelmingly, students expressed being very satisfied with their experiences within both the treatment and the control groups. Responses from each group will be shared separately by survey question.

Survey Question #1: "If a friend asked you about this math program, what 3 words would you use to describe the program?"

Control Group on Survey Question #1

In response to Question #1 on the Classroom Practices Survey, nearly every student in the control group included "fun" as one of their descriptors. The next most common descriptor was "interesting/exciting," and the third most common descriptor was "challenging/hard." No control group students included any negative descriptors of the program in response to Question 1#.

Treatment Group on Survey Question #1

Overwhelmingly, like the control group, in response to survey Question #1, the treatment group students described the math program as "fun." Nearly every student in the treatment group included "fun" as one of the three descriptors of the program that they were asked to list. The second most common descriptor of the program was "challenging/hard," and the third most common descriptor of the program was "helpful/useful."

While no control group students included any negative descriptors in response to Question #1 on the survey, a few treatment group students did. A few students described the program as "confusing," and a few used general statements about their enjoyment level of the program, such as "not so fun."

However, overall, treatment group students, like control group students, expressed a great deal of satisfaction with and enjoyment of the math program in which they participated.

Survey Question #2: "Describe the activity or activities that you did in this class that helped you learn the most math."

Control Group on Survey Question #2

In response to Question #2 on the Classroom Practices Survey, the most common responses of the control group students were, "graphing" and "tables" or a combination of the two. Numerous students also indicated that they learned best "when we learned how to do things with the calculator."

Treatment Group on Survey Question #2

The most common response of treatment group students to Question #2 was that using the graphing calculator was most helpful to them in learning math. Other students noted that doing the Equate puzzles was helpful to them. Other students were more general in their responses, indicating that "equations" were helpful to them, as was "graphing." A few students noted that the structure of the class was helpful to them in learning—"learning as a group" and "taking breaks" were both noted be several students as aiding in the learning process.

Survey Question #3: Describe the activity or activities that were *least* helpful to you in learning math.

Control Group on Survey Question #3

Control students were less in agreement about the activity or activities that were *least* helpful to them in learning math than they were about those activities that helped them. Many students indicated that there were no activities that were not helpful to them (e.g., "There were no activities that were *not* helpful"). Interestingly, the majority of students who cited a particular activity that was not helpful to them in learning math cited activities that they already knew how to do—in most instances, this was creating a graph on paper. One student wrote, "I would say the graph because I already knew half of the things." Another student wrote, "Graphing on paper, I already know how."

Treatment Group on Survey Question #3

Treatment group students were also varied in their responses to what activities were least helpful to them in learning math. Like the control group, many treatment group students indicated that all of the activities in the program were helpful to them. The most common negative response was that "turning the story into a graph" was not helpful. Other students noted that the number of tests they had to take during the program was not helpful to their learning. Other students noted that the book work was less helpful to them than hands-on activities.

Survey Question #4: How was this class different from your math classes at school?

Control Group on Survey Question #4

When asked how the summer class differed from their usual math classes at school, many control group students noted differences in the challenge level: the students perceived the summer program as moving at a faster pace, being more challenging, and involving greater levels of learning than their regular math classes. In addition to being more *challenging*, most control students also noted that the summer program was, for various reasons, more *enjoyable* than their usual math classes—for some students, they enjoyed "getting to talk more and having less math worksheets." Others enjoyed the smaller class sizes of the summer program and noted that they "got more help" and "individual attention" than they did during their regular math classes.

Treatment Group on Survey Question #4

Interestingly, while many control group students noted a difference between the challenge level offered in the summer program and that offered in their regular math classes, few students in the treatment group classes noted this difference. While many treatment group students included the descriptor of "challenging" in their list of words to describe the program in response to Survey Question #1, they did not use it when comparing it to their regular math classes. Those who did make comparisons of challenge noted the greater detail that the summer program went into. "It covered more detail for better learning," one student noted.

Treatment group students' responses focused more on the higher level of enjoyment they experienced within the summer program than in their regular math classes. Every student responding indicated that they preferred the summer program to their regular math class: "it was a lot more fun," "funner," "it was less stressful," "it was less work."

Many treatment group students also indicated that they preferred the summer class because "there was no homework." Another common response was that the summer class had fewer students in it and the teacher had more time to work individually with students. A few students also noted that their teacher was less stressed during the summer than during the school year: "my teacher is more relaxed and not mean. We had a good time."

Questions #5 & #6: What was most challenging about these math lessons? What was least challenging about these lessons?

Control Group on Survey Questions #5 & #6

The most common response to what control group students found most challenging in the summer program were "equations." A few students noted that the tests were the most challenging part of the program for them.

The most common response to what control group students found *least* challenging was "graphing." A few students also indicated that the "book work" was the least challenging aspect of the class.

Treatment Group on Survey Questions #5 & #6

Treatment group students were less unanimous in their responses to what they found most and least challenging, with more varied responses. However, like the control group students, what they found to be most challenging were "equations." A close second was "changing tables into graphs," and the third most common response was "using variables."

What the majority of treatment group students found least challenging was "graphing on the calculator."

Discussion

Although there were no statistically significant differences between treatment and control groups on achievement, aptitude, or attitudes, three important findings emerged from the qualitative data that merit consideration. Each will be considered separately below.

1. One interesting finding emerging from the study was that all teachers unanimously expressed liking being provided with a prescribed curriculum that was easy for them to follow. All perceived the curriculum to be high-level, challenging, and engaging for students, as well as enjoyable to teach. As a result, all teachers maintained a high level of fidelity to the treatments. This suggests that, to encourage treatment fidelity in studies asking teachers to implement a certain type of curriculum, a highly prescribed, scripted curriculum may be most effective.

2. Students in both the treatment and control groups expressed thoroughly enjoying the math program. Students from both groups cited the small class size, the "fun" and interactive math activities, and the high level of challenge as the primary reasons for enjoying the program. None mentioned the technological components as contributing to their enjoyment of or engagement in the program. This is interesting in light of recent attention focused in the literature on the use of technology to engage students in learning math. However, the students in this study—whether provided with technology as a learning tool or not—indicated that raising the challenge level of the content, providing hands-on activities, and providing more intimate learning environments with opportunities for one-on-one discussions with the teacher may be the keys to increased student engagement in and enjoyment of math.

   The treatment group students did indicate that the graphing calculators were useful tools in helping them to learn math. Control students also indicated that the calculators, when they were distributed to them at the end of the program, were helpful in learning math.

   Taken together, these two findings suggest that while technology provides a useful pathway to understanding for students, it alone does not necessarily encourage or ensure student engagement. Instead, it seems that for the students in this study at least, high-level challenge, one-on-one time with the teacher, and hands-on activities are what is needed to engage advanced students in learning math.

3. Students in the study indicated a clear preference for learning at a faster pace and at greater levels of challenge than they normally got the opportunity to do in their regular math classes. Nearly all of the participating students indicated that they learned *better* under the conditions of a quickened pace and increased challenge. Again, nearly all participating students indicated an eagerness to learn more math than they were able to do during their regular school year classes. This signals a need for a consideration of the match between the challenge level of the mathematics curriculum offered in our middle classrooms and the needs and abilities of the adolescents populating these classrooms. It begs the questions, *Are we underestimating the level of mathematical ability and interest of many of our middle school students? Are we limiting the possibilities for able math students by the lack of fit between the curriculum and instruction offered in our middle school math classes and their mathematical abilities and interests?*

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Appendix A
Unclogging the Mathematics Pipeline Through Access to Algebraic Understanding
Teacher's Log

Unclogging the Mathematics Pipeline Through Access to Algebraic Understanding Teacher's Log

Investigation # ________

Date: ______________________________ School: ____________________________

Teacher: ______________________________ Intervention: 1 (Connected Math)

BRIEFLY DESCRIBE YOUR APPROACH TO THE INVESTIGATION:

LAUNCH:
EXPLORE:
SUMMARIZE:
Please describe the students' reactions to this investigation:
List students who completed challenge ACEs

Unclogging the Mathematics Pipeline Through Access to Algebraic Understanding Teacher's Log

Investigation # _______

Date: ____________________________ School: __________________________

Teacher: ____________________________ Intervention: 2 (Technology) ____________

BRIEFLY DESCRIBE YOUR APPROACH TO THE INVESTIGATION:

LAUNCH:
EXPLORE:
SUMMARIZE:
USE OF TECHNOLOGY:
Please describe the students' reactions to this investigation:
List students who completed challenge ACEs

Appendix B
Teacher Interview Questions

TEACHER INTERVIEW QUESTIONS
MARK OLIVER
UNIVERSITY OF CONNECTICUT

Firstly, thank you for participating in this interview. We have noticed that the teachers in the project are particularly strong math instructors, and we therefore wished to interview each teacher to gather further information about his/her instructional practices. Some of the questions will require you to think about your concepts about teaching, and others will see you discuss your ideas about teaching high potential math students.

The following questions are designed to explore the impact of teacher factors on the effective instruction of high potential math students. The factors that are explored by this interview include:

A. Beliefs/self-efficacy about own math abilities (particularly algebra);
B. Personal epistemology regarding mathematic instruction (problem solving, constructivist approach, collaborative learning, etc);
C. Instructional efficacy (how confident the teachers feel in teaching the subject matter – mathematics in general, and specifically Connected Math); and
D. Beliefs about high potential math students (characteristics, instructional needs, etc).

QUESTIONS

1. During the initial training, you recalled your school experiences about learning algebra. Would you now describe the strongest memory that you possess concerning learning math at school? (A)

   Prompt: Think about the qualities of the teacher, the methods used to teach (e.g., group-work, problem solving, bookwork), and if the experience was positive or negative.

2. Do you believe that this experience has had an influence on the way that you currently teach your students? If yes, please explain? (B)

3. What do you consider to be the most important components of good math instruction? (B)

4. Think about how you use math skills in everyday life.

   - In what ways do you use math skills in everyday activities?
   - How confident do you feel in using math skills in everyday activities?
   - Would you like to improve your math skills? Please explain

5. Effective teaching and learning depend upon several teacher factors. In your opinion, what are the characteristics of an effective math teacher? (C)

6. Have you ever taught a high potential math student? Please describe your experience/s.

7. What do you believe are the characteristics of high potential math students? (D)

8. Are high potential math students easy to identify? Please explain. (D)

9. Are high potential math students easy to instruct? Please explain. (D)

10. What do you believe are the best methods for developing the talents of high potential math students? (D) (*instructional approaches, curriculum, grouping, mentoring etc.)*.

11. How confident do you feel in being able to cater to high potential math students? Please explain your response. (C)

12. To conclude this interview, I would like for you to think about the skills, knowledge and qualities of an effective teacher of high potential math students. I would like you to construct a picture (model/mindmap) that displays these qualities. You may wish to talk aloud as you draw your model.

    <u>Prompt</u>: Encourage the interviewee to explain his/her model as he/she draws, and add verbal comments/details that he/she has not included in the model).

Thank you once more for participating in this interview and for your continued enthusiasm.

Appendix C
Interview Questions for Principals

INTERVIEW QUESTIONS FOR PRINCIPALS
MARK OLIVER
UNIVERSITY OF CONNECTICUT

Firstly, thank you for participating in this interview.

I wished to spend some time with you to discuss the project implementation, particularly with regards to effective teaching and the instruction of high potential math students.

The following questions are designed to explore the impact of teacher factors on the effective instruction of high potential math students. The factors that are explored by this interview include:

A. Beliefs/self-efficacy about own math abilities (particularly algebra);
B. Personal epistemology regarding mathematic instruction (problem solving, constructivist approach, collaborative learning, etc);
C. Instructional efficacy (how confident the teachers feel in teaching the subject matter—mathematics in general, and specifically Connected Math); and
D. Beliefs about high potential math students (characteristics, instructional needs, etc).

QUESTIONS

1. What do you consider to be the most important components of good math instruction? (B)

2. Effective teaching and learning depends upon several teacher factors. In your opinion, what are the characteristics of an effective math teacher? (C)

3. Have you ever taught a high potential math student? Please describe your experience/s.

4. What do you believe are the characteristics of high potential math students? (D)

5. Are high potential math students easy to identify? Please explain. (D)

6. Are high potential math students easy to instruct? Please explain. (D)

7. What do you believe are the best methods for developing the talents of high potential math students? (D) (*instructional approaches, curriculum, grouping, mentoring etc).*

8. Further questions . . .

Appendix D
Mathematics Teacher Questionnaire

Name: ____

School: ____

Date: ____

MATH TEACHER QUESTIONNAIRE

Instructions: When answering the following questions, please refer only to your afterschool math class (not your regular math classes).

1. In a typical class period, what percentage of time do students spend on each of the following activities?

	Write in the percent The total should add to 100%
a. Reviewing assigned seatwork	___%
b. Listening to lecture-style presentations	___%
c. Working problems with your guidance	___%
d. Working problems on their own without your guidance	___%
e. Listening to you re-teach and clarify content/procedures	___%
f. Taking tests or quizzes	___%
g. Participating in classroom management tasks not related to the lesson's content/purpose (e.g., interruptions and keeping order)	___%
h. Other student activities	___%
i. Having snack time	___%
TOTAL	100%

2. When you assign seatwork to the students, about how many minutes do you usually assign? (Consider the time it would take an average student in your class)

0-5 minutes	6-10 minutes	11-15 minutes	16-20 minutes	> 20 minutes

3. **How often do you do the following with assigned seatwork?**

   a. Monitor whether or not the seatwork was completed

1	2	3	4	5
Never	Rarely	Sometimes	Often	Always

   b. Correct seatwork and then give feedback to students

1	2	3	4	5
Never	Rarely	Sometimes	Often	Always

   c. Have students correct their own seatwork in class

1	2	3	4	5
Never	Rarely	Sometimes	Often	Always

   d. Use seatwork as a basis for class discussion

1	2	3	4	5
Never	Rarely	Sometimes	Often	Always

4. **In your teaching, how often do you usually ask students to do the following?**

   a. Practice adding, subtracting, multiplying, and dividing

1	2	3	4	5
Never	Rarely	Sometimes	Often	Always

   b. Work on fractions and decimals

1	2	3	4	5
Never	Rarely	Sometimes	Often	Always

   c. Work on problems for which there is no immediately obvious method of solution

1	2	3	4	5
Never	Rarely	Sometimes	Often	Always

   d. Interpret data in tables, charts, or graphs

1	2	3	4	5
Never	Rarely	Sometimes	Often	Always

   e. Write equations and functions to represent relationships

1	2	3	4	5
Never	Rarely	Sometimes	Often	Always

   f. Work together in small groups

1	2	3	4	5
Never	Rarely	Sometimes	Often	Always

g. Relate what they are learning in mathematics to their daily lives

1	2	3	4	5
Never	Rarely	Sometimes	Often	Always

h. Explain their answers

1	2	3	4	5
Never	Rarely	Sometimes	Often	Always

i. Decide on their own procedures for solving complex problems

1	2	3	4	5
Never	Rarely	Sometimes	Often	Always

5. **In your view, to what extent do the following limit how you teach the class?**

a. Students with different academic abilities

1	2	3	4	5
Never	Rarely	Sometimes	Often	Always

b. Uninterested students

1	2	3	4	5
Never	Rarely	Sometimes	Often	Always

c. Low morale among students

1	2	3	4	5
Never	Rarely	Sometimes	Often	Always

d. Disruptive students

1	2	3	4	5
Never	Rarely	Sometimes	Often	Always

6. **How often do students use calculators for the following activities?**

a. Check their answers

1	2	3	4	5
Never	Rarely	Sometimes	Often	Always

b. Do routine computations

1	2	3	4	5
Never	Rarely	Sometimes	Often	Always

c. Solve complex problems

1	2	3	4	5
Never	Rarely	Sometimes	Often	Always

d. Explore number concepts

1	2	3	4	5
Never	Rarely	Sometimes	Often	Always

7. **How often do students use computers for the following activities?**

a. Discover mathematics principles and concepts

1	2	3	4	5
Never	Rarely	Sometimes	Often	Always

b. Practice skills and procedures

1	2	3	4	5
Never	Rarely	Sometimes	Often	Always

c. Look up ideas and information

1	2	3	4	5
Never	Rarely	Sometimes	Often	Always

8. **Please indicate your agreement or disagreement with each of the following statements:**

a. I feel comfortable using technology with my students.

1	2	3	4	5
Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree

b. I think it is important to use technology in my mathematics teaching.

1	2	3	4	5
Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree

c. Technology does not benefit students' learning of mathematics.

1	2	3	4	5
Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree

d. Students are more motivated to learn mathematics when technology is involved.

1	2	3	4	5
Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree

9. **Please indicate your agreement or disagreement with each of the following statements:**

   a. My students are rarely challenged by the math content in class.

1	2	3	4	5
Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree

   b. My students feel comfortable asking questions when they do not understand.

1	2	3	4	5
Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree

   c. My students think that mathematics is useful in everyday life.

1	2	3	4	5
Strongly Disagree	Disagree	Neutral	Agree	Strongly Agree

Appendix E
Unclogging the Mathematics Pipeline
Classroom Observation Scale

Unclogging the Mathematics Pipeline
Classroom Observation Scale

Teacher: ______________________ **Date:** ______________

Observer: ______________________ **Time observation began:** ______________

School: ______________________ **Time observation ended:** ______________

Program Teacher: ______________________ **Intervention 1 or 2:** ______________

Items	**Field Notes**
1. Provides clear and measurable objectives 1 Not effective — 2 Partially effective — 3 Moderately effective — 4 Very effective	
2. Ensures that students understand lessons and assignments 1 Not effective — 2 Partially effective — 3 Moderately effective — 4 Very effective	
3. Promotes connections to prior mathematical knowledge, skills, and understandings 1 Not effective — 2 Partially effective — 3 Moderately effective — 4 Very effective	
4. Uses a variety of tools to reason together about algebra 1 Not effective — 2 Partially effective — 3 Moderately effective — 4 Very effective	

5. Engages students in lessons

1	2	3	4
Not effective	Partially effective	Moderately effective	Very effective

6. Reflects on the students' reactions to lessons

1	2	3	4
Not effective	Partially effective	Moderately effective	Very effective

7. Asks questions to press the students onward with solving the algebra

1	2	3	4
Not effective	Partially effective	Moderately effective	Very effective

8. Promotes communication about mathematics

1	2	3	4
Not effective	Partially effective	Moderately effective	Very effective

9. Engages students' intellect

1	2	3	4
Not effective	Partially effective	Moderately effective	Very effective

10. Listens to students' comments and responses carefully to assess understanding

1	2	3	4
Not effective	Partially effective	Moderately effective	Very effective

11. Encourages discourse about mathematical problems

1	2	3	4
Not effective	Partially effective	Moderately effective	Very effective

12. Observes, listens to, and gathers information about students to assess their learning

1	2	3	4
Not effective	Partially effective	Moderately effective	Very effective

13. Assesses students' mathematical knowledge and understanding formally

1	2	3	4
Not effective	Partially effective	Moderately effective	Very effective

14. Encourages a positive disposition toward mathematics

1	2	3	4
Not effective	Partially effective	Moderately effective	Very effective

Appendix F
Mathematics Classroom Practices Survey
Algebra Research Study

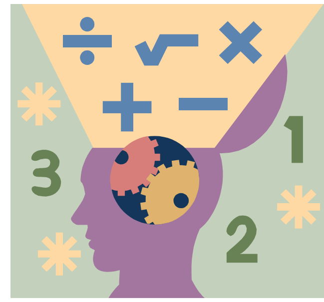
Mathematics Classroom Practices Survey
Algebra Research Study

Name: ___________________________________

School: __________________________________

Teacher: _________________________________

Date: ____________________________________



Part A:
Directions: Please write your comments in response to questions 1 to 6.

① If a friend asked you about this math program what 3 words would you use to describe the program?

❷ Describe the activity or activities that you did in this class that helped you learn the most math.

③ Describe the activity or activities that were ***least*** helpful to you in learning math.

❹ How was this class different from your math classes at school?

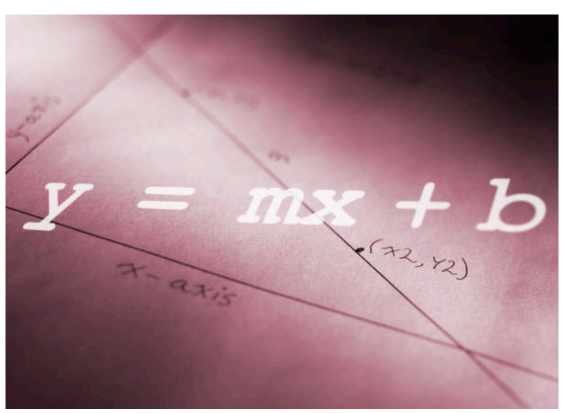
⑤ What was most challenging about these math lessons?

❻ What was least challenging about these lessons?

Part B:
Directions: Please circle one response to each of the questions below.

⑦ How good are you at math?
- a) I am a math whiz
- b) I do very good work in math
- c) I do average in math
- d) I struggle in math
- e) I cannot do math well at all

❽ How interesting were the math lessons?
- a) I thought the lessons were very interesting.
- b) I thought most lessons were interesting.
- c) I only thought some lessons were interesting.
- d) I thought some lessons were not very interesting.
- e) I thought that most lessons were not very interesting



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